

3MJS/3MJ

本科 1 期 4 月度

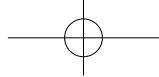
解答

Z会東大進学教室

中3数学

中3東大数学





1章 数と式 (1) 一式の展開ー

問題

【1】 (1) $-3x^2y^3 = -3y^3 \times x^2$ より,

係数は $-3y^3$, 次数は 2 次

(2) $5ab^2x = 5b^2x \times a$ より,

係数は $5b^2x$, 次数は 1 次

(3) $xy^3z^2 = xz^2 \times y^3$ より,

係数は xz^2 , 次数は 3 次

(4) $-\frac{2x^3yz^4}{3} = -\frac{2x^3y}{3} \times z^4$ より,

係数は $-\frac{2x^3y}{3}$, 次数は 4 次

【2】 (1) $x^2 - 2xy + 5y = (-2x + 5)y + x^2$

より, 次数は 1 次, 定数項は x^2

(2) $1 - 2x + xy^3 - 2x^2y + 5x = 1 + 3x + xy^3 - 2x^2y$

$$= -2yx^2 + (3 + y^3)x + 1$$

より, 次数は 2 次, 定数項は 1

(3) $4x^5 - 2x^3 + 3xy^2 - y + 2 = 3xy^2 - y + 4x^5 - 2x^3 + 2$

より, 次数は 2 次, 定数項は $4x^5 - 2x^3 + 2$

(4) $a^2x - by^2 + a^2 - 3axy - b^2x^3 + by^2 - y + 2$

$$= a^2x + a^2 - 3axy - b^2x^3 - y + 2$$

$$= -b^2x^3 + (a^2 - 3ay)x + a^2 - y + 2$$

より, 次数は 3 次, 定数項は $a^2 - y + 2$

【3】 (1) $2x - 5x^3 + 3 - x^2 = -5x^3 - x^2 + 2x + 3$

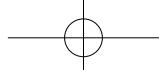
(2) $3x^2y - y + 2x^2 + 7xy = (3y + 2)x^2 + 7yx - y$

(3) $3 - x^2 + 2x - 5 + x^2 - x = x - 2$

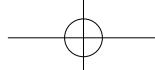
(4) $5ax + ab - 3bx + 2x^3 + ax - x^2 + 5bx - 1$

$$= 6ax + ab + 2bx + 2x^3 - x^2 - 1$$

$$= 2x^3 - x^2 + (6a + 2b)x + ab - 1$$



- 【4】 (1) $3a^2b(a - b + 1) = \mathbf{3a^3b - 3a^2b^2 + 3a^2b}$
- (2) $3xy^2(3x - y + 2) = \mathbf{9x^2y^2 - 3xy^3 + 6xy^2}$
- (3) $3ab^2(a^2 - 2ab + 3b^2) = \mathbf{3a^3b^2 - 6a^2b^3 + 9ab^4}$
- (4) $-2ab^2c(3a - 2b + c) = \mathbf{-6a^2b^2c + 4ab^3c - 2ab^2c^2}$
- (5) $(x + 2)(x^2 - 6x + 5) = x(x^2 - 6x + 5) + 2(x^2 - 6x + 5)$
 $= x^3 - 6x^2 + 5x + 2x^2 - 12x + 10$
 $= \mathbf{x^3 - 4x^2 - 7x + 10}$
- (6) $(x - 1)(3x^2 + 2x - 7) = x(3x^2 + 2x - 7) - (3x^2 + 2x - 7)$
 $= 3x^3 + 2x^2 - 7x - 3x^2 - 2x + 7$
 $= \mathbf{3x^3 - x^2 - 9x + 7}$
- (7) $(5x - y)(4x^2 - xy + 6y^2) = 5x(4x^2 - xy + 6y^2) - y(4x^2 - xy + 6y^2)$
 $= 20x^3 - 5x^2y + 30xy^2 - 4x^2y + xy^2 - 6y^3$
 $= \mathbf{20x^3 - 9x^2y + 31xy^2 - 6y^3}$
- (8) $(a - 2b)(6a^2 - ab + 9b^2) = a(6a^2 - ab + 9b^2) - 2b(6a^2 - ab + 9b^2)$
 $= 6a^3 - a^2b + 9ab^2 - 12a^2b + 2ab^2 - 18b^3$
 $= \mathbf{6a^3 - 13a^2b + 11ab^2 - 18b^3}$
- (9) $(x + y - 3)(3x - 2y + 1)$
 $= x(3x - 2y + 1) + y(3x - 2y + 1) - 3(3x - 2y + 1)$
 $= 3x^2 - 2xy + x + 3xy - 2y^2 + y - 9x + 6y - 3$
 $= \mathbf{3x^2 + xy - 8x - 2y^2 + 7y - 3}$
- (10) $(x^2 - 2x + 3)(2x^3 - 3x^2 + 1)$
 $= x^2(2x^3 - 3x^2 + 1) - 2x(2x^3 - 3x^2 + 1) + 3(2x^3 - 3x^2 + 1)$
 $= 2x^5 - 3x^4 + x^2 - 4x^4 + 6x^3 - 2x + 6x^3 - 9x^2 + 3$
 $= \mathbf{2x^5 - 7x^4 + 12x^3 - 8x^2 - 2x + 3}$
- (11) $(a^2 - ab + 2b^2)(a^2 + ab - 3b^2)$
 $= a^2(a^2 + ab - 3b^2) - ab(a^2 + ab - 3b^2) + 2b^2(a^2 + ab - 3b^2)$
 $= a^4 + a^3b - 3a^2b^2 - a^3b - a^2b^2 + 3ab^3 + 2a^2b^2 + 2ab^3 - 6b^4$
 $= \mathbf{a^4 - 2a^2b^2 + 5ab^3 - 6b^4}$
- (12) $(x^2 + 1 - x)(2x^2 - x - 3)$
 $= (x^2 - x + 1)(2x^2 - x - 3)$
 $= x^2(2x^2 - x - 3) - x(2x^2 - x - 3) + (2x^2 - x - 3)$
 $= 2x^4 - x^3 - 3x^2 - 2x^3 + x^2 + 3x + 2x^2 - x - 3$
 $= \mathbf{2x^4 - 3x^3 + 2x - 3}$



【5】(1) x^3 の項の係数は、係数だけ計算すると、

$$\begin{aligned}1 \times 2 - 7 \times (-3) &= 2 + 21 \\&= \mathbf{23}\end{aligned}$$

(2) x^4 の項の係数は、係数だけ計算すると、

$$\begin{aligned}2 \times 4 + 6 \times 3 &= 8 + 18 \\&= \mathbf{26}\end{aligned}$$

(3) x^5 の項の係数は、係数だけ計算すると、

$$\begin{aligned}7 \times 2 + 12 \times 3 - 3 \times 1 &= 14 + 36 - 3 \\&= \mathbf{47}\end{aligned}$$

(4) x^4 の項の係数は、係数だけ計算すると、

$$\begin{aligned}5 \times 3 - 6 \times (-6) + 3 \times 5 &= 15 + 36 + 15 \\&= \mathbf{66}\end{aligned}$$

(5) x^3 の項の係数は、係数だけ計算すると、

$$\begin{aligned}1 \times (-1) \times (-1) + 1 \times (-2) \times 2 + 1 \times 1 \times (-1) + 1 \times (-1) \times 2 \\+ 1 \times (-2) \times 1 + 1 \times 1 \times 2 + 1 \times (-1) \times 1 \\= 1 - 4 - 1 - 2 - 2 + 2 - 1 \\= \mathbf{-7}\end{aligned}$$

【6】(1) $(3x + 2)^3 = (3x)^3 + 3 \cdot (3x)^2 \cdot 2 + 3 \cdot 3x \cdot 2^2 + 2^3$

$$= 3^3 x^3 + 3 \cdot 3^2 x^2 \cdot 2 + 3 \cdot 3x \cdot 4 + 8$$

$$= 27x^3 + 3 \cdot 9x^2 \cdot 2 + 36x + 8$$

$$= \mathbf{27x^3 + 54x^2 + 36x + 8}$$

(2) $(2a - 4)^3 = (2a)^3 - 3 \cdot (2a)^2 \cdot 4 + 3 \cdot 2a \cdot 4^2 - 4^3$

$$= 2^3 a^3 - 3 \cdot 2^2 a^2 \cdot 4 + 3 \cdot 2a \cdot 16 - 64$$

$$= 8a^3 - 3 \cdot 4a^2 \cdot 4 + 96a - 64$$

$$= \mathbf{8a^3 - 48a^2 + 96a - 64}$$

(3) $(x + 5y)^3 = x^3 + 3 \cdot x^2 \cdot 5y + 3 \cdot x \cdot (5y)^2 + (5y)^3$

$$= x^3 + 15x^2y + 3 \cdot x \cdot 5^2 y^2 + 5^3 y^3$$

$$= x^3 + 15x^2y + 3 \cdot x \cdot 25y^2 + 125y^3$$

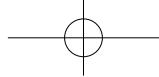
$$= \mathbf{x^3 + 15x^2y + 75xy^2 + 125y^3}$$

(4) $(3a - 4b)^3 = (3a)^3 - 3 \cdot (3a)^2 \cdot 4b + 3 \cdot 3a \cdot (4b)^2 - (4b)^3$

$$= 3^3 a^3 - 3 \cdot 3^2 a^2 \cdot 4b + 3 \cdot 3a \cdot 4^2 b^2 - 4^3 b^3$$

$$= 27a^3 - 3 \cdot 9a^2 \cdot 4b + 3 \cdot 3a \cdot 16b^2 - 64b^3$$

$$= \mathbf{27a^3 - 108a^2b + 144ab^2 - 64b^3}$$



$$(5) \quad (x+1)(x^2 - x + 1) = x^3 + 1^3 \\ = x^3 + 1$$

$$(6) \quad (3a-2)(9a^2 + 6a + 4) = (3a)^3 - 2^3 \\ = 3^3 a^3 - 8 \\ = 27a^3 - 8$$

$$(7) \quad (2a-3b)(4a^2 + 6ab + 9b^2) = (2a)^3 - (3b)^3 \\ = 2^3 a^3 - 3^3 b^3 \\ = 8a^3 - 27b^3$$

$$(8) \quad (x-2y)(x^2 + 2xy + 4y^2) = x^3 - (2y)^3 \\ = x^3 - 2^3 y^3 \\ = x^3 - 8y^3$$

$$(9) \quad (x+y+1)^2 = x^2 + y^2 + 1^2 + 2 \cdot x \cdot y + 2 \cdot y \cdot 1 + 2 \cdot 1 \cdot x \\ = x^2 + y^2 + 1 + 2xy + 2y + 2x \\ = x^2 + y^2 + 2xy + 2x + 2y + 1$$

$$(10) \quad (a+b-2)^2 = a^2 + b^2 + (-2)^2 + 2 \cdot a \cdot b + 2 \cdot b \cdot (-2) + 2 \cdot (-2) \cdot a \\ = a^2 + b^2 + 4 + 2ab - 4b - 4a \\ = a^2 + b^2 + 2ab - 4a - 4b + 4$$

$$(11) \quad (2x+y-z)^2 = (2x)^2 + y^2 + (-z)^2 + 2 \cdot 2x \cdot y + 2 \cdot y \cdot (-z) + 2 \cdot (-z) \cdot 2x \\ = 2^2 \cdot x^2 + y^2 + z^2 + 4xy - 2yz - 4xz \\ = 4x^2 + y^2 + z^2 + 4xy - 2yz - 4xz$$

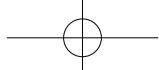
$$(12) \quad (a-2b-3c)^2 \\ = a^2 + (-2b)^2 + (-3c)^2 + 2 \cdot a \cdot (-2b) + 2 \cdot (-2b) \cdot (-3c) + 2 \cdot (-3c) \cdot a \\ = a^2 + 4b^2 + 9c^2 - 4ab + 12bc - 6ca$$

【7】 (1)

$$\begin{aligned} & (x+1)(x-1)(x^2 + x + 1)(x^2 - x + 1) \\ &= \{(x+1)(x^2 - x + 1)\}\{(x-1)(x^2 + x + 1)\} \\ &= (x^3 + 1)(x^3 - 1) \\ &= (x^3)^2 - 1 \\ &= x^{3 \times 2} - 1 \\ &= x^6 - 1 \end{aligned}$$

(2)

$$\begin{aligned} & (a-1)(a-2)(a^2 + a + 1)(a^2 + 2a + 4) \\ &= \{(a-1)(a^2 + a + 1)\}\{(a-2)(a^2 + 2a + 4)\} \\ &= (a^3 - 1)(a^3 - 8) \\ &= (a^3)^2 - 9a^3 + 8 \\ &= a^{3 \times 2} - 9a^3 + 8 \\ &= a^6 - 9a^3 + 8 \end{aligned}$$



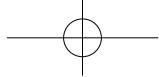
$$\begin{aligned}(3) \quad & (x-y)(x^2+xy+y^2)(x^6+x^3y^3+y^6) \\& =\{(x-y)(x^2+xy+y^2)\}(x^6+x^3y^3+y^6) \\& =(x^3-y^3)(x^6+x^3y^3+y^6) \\& =(x^3)^3-(y^3)^3 \\& =x^{3\times 3}-y^{3\times 3} \\& =\mathbf{x^9 - y^9}\end{aligned}$$

$$\begin{aligned}(4) \quad & (a+1)(a-1)(a^6-4)(a^4+a^2+1) \\& =\{(a+1)(a-1)\}(a^6-4)(a^4+a^2+1) \\& =(a^2-1)(a^6-4)(a^4+a^2+1) \\& =\{(a^2-1)(a^4+a^2+1)\}(a^6-4) \\& =\{(a^2)^3-1^3\}(a^6-4) \\& =(a^{2\times 3}-1)(a^6-4) \\& =(a^6-1)(a^6-4) \\& =(a^6)^2-5a^6+4 \\& =a^{6\times 2}-5a^6+4 \\& =\mathbf{a^{12} - 5a^6 + 4}\end{aligned}$$

$$\begin{aligned}(5) \quad & (x-1)^2(x+1)^2(x^2+1)^2=\{(x-1)(x+1)(x^2+1)\}^2 \\& =\{(x^2-1)(x^2+1)\}^2 \\& =\{(x^2)^2-1^2\}^2 \\& =(x^{2\times 2}-1)^2 \\& =(x^4-1)^2 \\& =(x^4)^2-2x^4+1 \\& =x^{4\times 2}-2x^4+1 \\& =\mathbf{x^8 - 2x^4 + 1}\end{aligned}$$

$$\begin{aligned}(6) \quad & (a-2)^2(a^2+2a+4)^2=\{(a-2)(a^2+2a+4)\}^2 \\& =(a^3-2^3)^2 \\& =(a^3-8)^2 \\& =(a^3)^2-16a^3+64 \\& =a^{3\times 2}-16a^3+64 \\& =\mathbf{a^6 - 16a^3 + 64}\end{aligned}$$

$$\begin{aligned}(7) \quad & (x+1)^3(x-1)^3=\{(x+1)(x-1)\}^3 \\& =(x^2-1)^3 \\& =(x^2)^3-3\cdot(x^2)^2\cdot 1+3\cdot x^2\cdot 1^2-1^3 \\& =x^{2\times 3}-3x^{2\times 2}+3x^2-1 \\& =\mathbf{x^6 - 3x^4 + 3x^2 - 1}\end{aligned}$$



$$\begin{aligned}
 (8) \quad (a+b)^2(a-b)^2(a^4+a^2b^2+b^4)^2 &= \{(a+b)(a-b)(a^4+a^2b^2+b^4)\}^2 \\
 &= \{(a^2-b^2)(a^4+a^2b^2+b^4)\}^2 \\
 &= \{(a^2)^3-(b^2)^3\}^2 \\
 &= (a^{2\times 3}-b^{2\times 3})^2 \\
 &= (a^6-b^6)^2 \\
 &= (a^6)^2-2a^6b^6+(b^6)^2 \\
 &= a^{6\times 2}-2a^6b^6+b^{6\times 2} \\
 &= \mathbf{a^{12}-2a^6b^6+b^{12}}
 \end{aligned}$$

[8] (1)
$$\begin{aligned}
 &(2x^2-x+3)^2 \\
 &= (2x^2)^2 + (-x)^2 + 3^2 + 2 \cdot 2x^2 \cdot (-x) + 2 \cdot (-x) \cdot 3 + 2 \cdot 3 \cdot 2x^2 \\
 &= 4x^4 + x^2 + 9 - 4x^3 - 6x + 12x^2 \\
 &= \mathbf{4x^4-4x^3+13x^2-6x+9}
 \end{aligned}$$

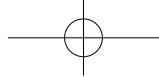
$$\begin{aligned}
 (2) \quad (a+b)^3(a^2-ab+b^2)^3 &= \{(a+b)(a^2-ab+b^2)\}^3 \\
 &= (a^3+b^3)^3 \\
 &= (a^3)^3 + 3 \cdot (a^3)^2 \cdot b^3 + 3 \cdot a^3 \cdot (b^3)^2 + (b^3)^3 \\
 &= a^{3\times 3} + 3a^{3\times 2}b^3 + 3a^3b^{3\times 2} + b^{3\times 3} \\
 &= \mathbf{a^9+3a^6b^3+3a^3b^6+b^9}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad (xy+z)^3 &= (xy)^3 + 3 \cdot (xy)^2 \cdot z + 3 \cdot xy \cdot z^2 + z^3 \\
 &= x^3y^3 + 3x^2y^2z + 3xyz^2 + z^3
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad (x^2-2x+3)(x^2+2x-3) &= \{x^2-(2x-3)\}\{x^2+(2x-3)\} \\
 &= (x^2)^2 - (2x-3)^2 \\
 &= x^4 - \{(2x)^2 - 12x + 9\} \\
 &= x^4 - (4x^2 - 12x + 9) \\
 &= \mathbf{x^4-4x^2+12x-9}
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad (a^3+a^2+a+1)(a^3-a^2+a-1) &= (a^3+a+a^2+1)(a^3+a-a^2-1) \\
 &= \{(a^3+a)+(a^2+1)\}\{(a^3+a)-(a^2+1)\} \\
 &= (a^3+a)^2 - (a^2+1)^2 \\
 &= (a^3)^2 + 2 \cdot a^3 \cdot a + a^2 - \{(a^2)^2 + 2a^2 + 1\} \\
 &= a^6 + 2a^4 + a^2 - (a^4 + 2a^2 + 1) \\
 &= a^6 + 2a^4 + a^2 - a^4 - 2a^2 - 1 \\
 &= \mathbf{a^6+a^4-a^2-1}
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad (25x^2+15xy+9y^2)(5x-3y) &= (5x)^3 - (3y)^3 \\
 &= \mathbf{125x^3-27y^3}
 \end{aligned}$$



$$\begin{aligned}
 (7) \quad & (x^3 + x^2 + x - 1)^2 \\
 & = \{(x^3 + x^2) + (x - 1)\}^2 \\
 & = (x^3 + x^2)^2 + 2(x^3 + x^2)(x - 1) + (x - 1)^2 \\
 & = (x^3)^2 + 2 \cdot x^3 \cdot x^2 + (x^2)^2 + 2(x^3 \cdot x - x^3 + x^2 \cdot x - x^2) + x^2 - 2x + 1 \\
 & = x^6 + 2x^5 + x^4 + 2(x^4 - x^3 + x^3 - x^2) + x^2 - 2x + 1 \\
 & = x^6 + 2x^5 + x^4 + 2(x^4 - x^2) + x^2 - 2x + 1 \\
 & = x^6 + 2x^5 + x^4 + 2x^4 - 2x^2 + x^2 - 2x + 1 \\
 & = x^6 + 2x^5 + 3x^4 - x^2 - 2x + 1
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad & (4x - 3y)^3 = (4x)^3 - 3 \cdot (4x)^2 \cdot 3y + 3 \cdot 4x \cdot (3y)^2 - (3y)^3 \\
 & = 64x^3 - 3 \cdot 16x^2 \cdot 3y + 3 \cdot 4x \cdot 9y^2 - 27y^3 \\
 & = 64x^3 - 144x^2y + 108xy^2 - 27y^3
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad & (a^2 + b^2)(a^4 - a^2b^2 + b^4) = (a^2)^3 + (b^2)^3 \\
 & = a^6 + b^6
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & (a^6 - 8a^3 + 64)(a^2 - 2a + 4)(a + 2) \\
 & = \{(a + 2)(a^2 - 2a + 4)\}(a^6 - 8a^3 + 64) \\
 & = (a^3 + 2^3)(a^6 - 8a^3 + 64) \\
 & = (a^3 + 8)(a^6 - 8a^3 + 64) \\
 & = (a^3)^3 + 8^3 \\
 & = a^9 + 512
 \end{aligned}$$

【9】(1) 左辺を展開して整理すると,

$$\begin{aligned}
 a(x - 2)^2 + b(x + 3) + c &= x^2 - x - 2 \\
 a(x^2 - 4x + 4) + bx + 3b + c &= x^2 - x - 2 \\
 ax^2 - 4ax + 4a + bx + 3b + c &= x^2 - x - 2 \\
 ax^2 + (-4a + b)x + 4a + 3b + c &= x^2 - x - 2
 \end{aligned}$$

両辺の係数を比較して,

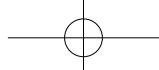
$$\begin{cases} a = 1 & \cdots ① \\ -4a + b = -1 & \cdots ② \\ 4a + 3b + c = -2 & \cdots ③ \end{cases}$$

①を②に代入すると

$$\begin{aligned}
 -4 \cdot 1 + b &= -1 \\
 -4 + b &= -1 \\
 \therefore b &= 3 \cdots ④
 \end{aligned}$$

①, ④を③に代入すると,

$$\begin{aligned}
 4 \cdot 1 + 3 \cdot 3 + c &= -2 \\
 4 + 9 + c &= -2 \\
 13 + c &= -2 \\
 \therefore c &= -15
 \end{aligned}$$



よって、

$$a = 1, b = 3, c = -15$$

<別解>

x についての恒等式ならば、どんな x についても等式が成り立つから、
 $x = 0$ を代入すると、

$$\begin{aligned} a(0-2)^2 + b(0+3) + c &= 0^2 - 0 - 2 \\ 4a + 3b + c &= -2 \end{aligned}$$

$x = 2$ を代入すると、

$$\begin{aligned} a(2-2)^2 + b(2+3) + c &= 2^2 - 2 - 2 \\ 5b + c &= 0 \end{aligned}$$

$x = -3$ を代入すると、

$$\begin{aligned} a(-3-2)^2 + b(-3+3) + c &= (-3)^2 - (-3) - 2 \\ 25a + c &= 10 \end{aligned}$$

となる。これを整理すると、

$$\begin{cases} 4a + 3b + c = -2 & \cdots ① \\ 5b + c = 0 & \cdots ② \\ 25a + c = 10 & \cdots ③ \end{cases}$$

$$\textcircled{2} \text{より}, 5b = -c \cdots \textcircled{4}$$

$$\textcircled{3} \text{より}, 25a = 10 - c \cdots \textcircled{5}$$

これを $\textcircled{1} \times 25$ に代入すると、

$$\begin{aligned} 100a + 75b + 25c &= -50 \\ 4 \cdot 25a + 15 \cdot 5b + 25c &= -50 \\ 4(10 - c) + 15 \cdot (-c) + 25c &= -50 \\ 40 - 4c - 15c + 25c &= -50 \\ 40 + 6c &= -50 \\ 6c &= -90 \\ c &= -15 \end{aligned}$$

これを $\textcircled{4}$ に代入して、

$$5b = -(-15)$$

$$5b = 15$$

$$\therefore b = 3$$

さらに $\textcircled{5}$ を代入して、

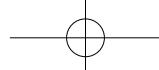
$$25a = 10 - (-15)$$

$$25a = 25$$

$$\therefore a = 1$$

を得る。逆を確かめる。

$$\begin{aligned} 1 \cdot (x-2)^2 + 3(x+3) + (-15) &= (x-2)^2 + 3(x+3) - 15 \\ &= x^2 - 4x + 4 + 3x + 9 - 15 \\ &= x^2 - x - 2 = \text{右辺} \end{aligned}$$



となり、確かに x についての恒等式である、よって、

$$a = 1, b = 3, c = -15$$

(2) 右辺を展開して整理すると、

$$\begin{aligned}4x^2 + 3x + 2 &= a(x-1)^2 + b(x-1) + c \\4x^2 + 3x + 2 &= a(x^2 - 2x + 1) + bx - b + c \\4x^2 + 3x + 2 &= ax^2 - 2ax + a + bx - b + c \\4x^2 + 3x + 2 &= ax^2 + (-2a+b)x + a - b + c\end{aligned}$$

両辺の係数を比較して、

$$\begin{cases} a = 4 & \cdots ① \\ -2a + b = 3 & \cdots ② \\ a - b + c = 2 & \cdots ③ \end{cases}$$

①を②に代入すると、

$$\begin{aligned}-2 \cdot 4 + b &= 3 \\-8 + b &= 3 \\\therefore b &= 11 \cdots ④\end{aligned}$$

①, ④を③に代入すると、

$$\begin{aligned}4 - 11 + c &= 2 \\-7 + c &= 2 \\\therefore c &= 9\end{aligned}$$

よって、

$$a = 4, b = 11, c = 9$$

<別解>

x についての恒等式ならば、どんな x についても等式が成り立つから、
 $x = 0$ を代入すると、

$$\begin{aligned}4 \cdot 0^2 + 3 \cdot 0 + 2 &= a(0-1)^2 + b(0-1) + c \\2 &= a - b + c\end{aligned}$$

$x = 1$ を代入すると、

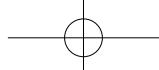
$$\begin{aligned}4 \cdot 1^2 + 3 \cdot 1 + 2 &= a(1-1)^2 + b(1-1) + c \\4 + 3 + 2 &= c \\9 &= c\end{aligned}$$

$x = 2$ を代入すると、

$$\begin{aligned}4 \cdot 2^2 + 3 \cdot 2 + 2 &= a(2-1)^2 + b(2-1) + c \\16 + 6 + 2 &= a + b + c \\24 &= a + b + c\end{aligned}$$

となる。これを整理すると、

$$\begin{cases} a - b + c = 2 & \cdots ① \\ c = 9 & \cdots ② \\ a + b + c = 24 & \cdots ③ \end{cases}$$



②を①に代入すると,

$$\begin{aligned} a - b + 9 &= 2 \\ a - b &= -7 \quad \cdots \textcircled{4} \end{aligned}$$

②を③に代入すると,

$$\begin{aligned} a + b + 9 &= 24 \\ a + b &= 15 \quad \cdots \textcircled{5} \end{aligned}$$

④+⑤より,

$$2a = 8 \quad \therefore a = 4$$

これを⑤に代入して,

$$4 + b = 15 \quad \therefore b = 11$$

を得る. 逆を確かめる.

$$\begin{aligned} 4(x-1)^2 + 11(x-1) + 9 &= 4(x^2 - 2x + 1) + 11x - 11 + 9 \\ &= 4x^2 - 8x + 4 + 11x - 11 + 9 \\ &= 4x^2 + 3x + 2 = \text{左辺} \end{aligned}$$

となり, 確かに x についての恒等式である. よって,

$$a = 4, b = 11, c = 9$$

(3) 左辺を展開して整理すると,

$$\begin{aligned} ax(x+1) + bx(x-1) + c(x-1)(x-3) &= x^2 - 3 \\ ax^2 + ax + bx^2 - bx + c(x^2 - 4x + 3) &= x^2 - 3 \\ ax^2 + ax + bx^2 - bx + cx^2 - 4cx + 3c &= x^2 - 3 \\ (a+b+c)x^2 + (a-b-4c)x + 3c &= x^2 - 3 \end{aligned}$$

両辺の係数を比較して,

$$\begin{cases} a+b+c = 1 & \cdots \textcircled{1} \\ a-b-4c = 0 & \cdots \textcircled{2} \\ 3c = -3 & \cdots \textcircled{3} \end{cases}$$

③より,

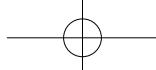
$$\begin{aligned} 3c &= -3 \\ c &= -1 \quad \cdots \textcircled{4} \end{aligned}$$

④を①に代入すると,

$$\begin{aligned} a + b - 1 &= 1 \\ a + b &= 2 \quad \cdots \textcircled{5} \end{aligned}$$

④を②に代入すると,

$$\begin{aligned} a - b - 4 \cdot (-1) &= 0 \\ a - b + 4 &= 0 \\ a - b &= -4 \quad \cdots \textcircled{6} \end{aligned}$$



⑤ + ⑥ より,

$$2a = -2 \quad \therefore a = -1$$

これを⑤に代入すると,

$$-1 + b = 2 \quad \therefore b = 3$$

よって,

$$a = -1, b = 3, c = -1$$

<別解>

x についての恒等式ならば、どんな x についても等式が成り立つから、
 $x = 0$ を代入すると,

$$\begin{aligned} a \cdot 0 \cdot (0+1) + b \cdot 0 \cdot (0-1) + c(0-1)(0-3) &= 0^2 - 3 \\ 3c &= -3 \\ c &= -1 \quad \cdots \textcircled{1} \end{aligned}$$

$x = 1$ を代入すると,

$$\begin{aligned} a \cdot 1 \cdot (1+1) + b \cdot 1 \cdot (1-1) + c(1-1)(1-3) &= 1^2 - 3 \\ 2a &= -2 \\ a &= -1 \quad \cdots \textcircled{2} \end{aligned}$$

$x = 3$ を代入すると,

$$\begin{aligned} a \cdot 3 \cdot (3+1) + b \cdot 3 \cdot (3-1) + c(3-1)(3-3) &= 3^2 - 3 \\ 12a + 6b &= 6 \\ 2a + b &= 1 \quad \cdots \textcircled{3} \end{aligned}$$

となる。②を③に代入すると,

$$\begin{aligned} 2 \cdot (-1) + b &= 1 \\ -2 + b &= 1 \\ \therefore b &= 3 \end{aligned}$$

を得る。逆を確かめる。

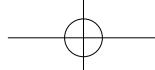
$$\begin{aligned} &-1 \cdot x(x+1) + 3x(x-1) + (-1) \cdot (x-1)(x-3) \\ &= -x^2 - x + 3x^2 - 3x - (x^2 - 4x + 3) \\ &= -x^2 - x + 3x^2 - 3x - x^2 + 4x - 3 \\ &= x^2 - 3 \\ &= \text{右辺} \end{aligned}$$

となり、確かに x についての恒等式である。よって,

$$a = -1, b = 3, c = -1$$

(4) 右辺を展開して整理すると,

$$\begin{aligned} x^2 + x + 1 &= a(x-1)(x-2) + b(x-2)(x-3) + c(x-3)(x-1) \\ x^2 + x + 1 &= a(x^2 - 3x + 2) + b(x^2 - 5x + 6) + c(x^2 - 4x + 3) \\ x^2 + x + 1 &= ax^2 - 3ax + 2a + bx^2 - 5bx + 6b + cx^2 - 4cx + 3c \\ x^2 + x + 1 &= (a+b+c)x^2 + (-3a-5b-4c)x + 2a + 6b + 3c \end{aligned}$$



両辺の係数を比較して、

$$\begin{cases} a+b+c=1 & \cdots \textcircled{1} \\ -3a-5b-4c=1 & \cdots \textcircled{2} \\ 2a+6b+3c=1 & \cdots \textcircled{3} \end{cases}$$

$$\textcircled{1} \times 4 + \textcircled{2} \text{ より}, \quad a-b=5 \quad \cdots \textcircled{4}$$

$$\textcircled{1} \times 3 - \textcircled{3} \text{ より}, \quad a-3b=2 \quad \cdots \textcircled{5}$$

$$\textcircled{4} - \textcircled{5} \text{ より},$$

$$2b=3 \quad \therefore b=\frac{3}{2} \quad \cdots \textcircled{6}$$

⑥を④を代入して、

$$a-\frac{3}{2}=5 \quad \therefore a=\frac{13}{2} \quad \cdots \textcircled{7}$$

⑥, ⑦を①に代入すると、

$$\begin{aligned} \frac{13}{2}+\frac{3}{2}+c &= 1 \\ 8+c &= 1 \\ \therefore c &= -7 \end{aligned}$$

よって、

$$a=\frac{13}{2}, \quad b=\frac{3}{2}, \quad c=-7$$

<別解>

x についての恒等式ならば、どんな x についても等式が成り立つから、 $x=1$ を代入すると、

$$\begin{aligned} 1^2+1+1 &= a(1-1)(1-2)+b(1-2)(1-3)+c(1-3)(1-1) \\ 3 &= 2b \\ \frac{3}{2} &= b \end{aligned}$$

$x=2$ を代入すると、

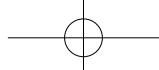
$$\begin{aligned} 2^2+2+1 &= a(2-1)(2-2)+b(2-2)(2-3)+c(2-3)(2-1) \\ 7 &= -c \\ -7 &= c \end{aligned}$$

$x=3$ を代入すると、

$$\begin{aligned} 3^2+3+1 &= a(3-1)(3-2)+b(3-2)(3-3)+c(3-3)(3-1) \\ 13 &= 2a \\ \frac{13}{2} &= a \end{aligned}$$

を得る。逆を確かめる。

$$\begin{aligned} &\frac{13}{2}(x-1)(x-2)+\frac{3}{2}(x-2)(x-3)-7(x-3)(x-1) \\ &= \frac{13}{2}(x^2-3x+2)+\frac{3}{2}(x^2-5x+6)-7(x^2-4x+3) \\ &= \frac{13}{2}x^2-\frac{39}{2}x+13+\frac{3}{2}x^2-\frac{15}{2}x+9-7x^2+28x-21 \\ &= x^2+x+1 \\ &= \text{左辺} \end{aligned}$$



となり、確かに x についての恒等式である。よって、

$$a = \frac{13}{2}, b = \frac{3}{2}, c = -7$$

(5) 両辺を展開して整理すると、

$$\begin{aligned}\text{左辺} &= (2x-1)^3 = (2x)^3 - 3 \cdot (2x)^2 \cdot 1 + 3 \cdot 2x \cdot 1^2 - 1^3 \\ &= 2^3 \cdot x^3 - 3 \cdot 2^2 \cdot x^2 \cdot 1 + 3 \cdot 2x \cdot 1 - 1 \\ &= 8x^3 - 12x^2 + 6x - 1\end{aligned}$$

$$\begin{aligned}\text{右辺} &= a(x-1)^3 + b(x-1)^2 + c(x-1) + d \\ &= a(x^3 - 3x^2 + 3x - 1) + b(x^2 - 2x + 1) + cx - c + d \\ &= ax^3 - 3ax^2 + 3ax - a + bx^2 - 2bx + b + cx - c + d \\ &= ax^3 + (-3a+b)x^2 + (3a-2b+c)x - a + b - c + d\end{aligned}$$

両辺の係数を比較して、

$$\begin{cases} a = 8 & \dots (1) \\ -3a + b = -12 & \dots (2) \\ 3a - 2b + c = 6 & \dots (3) \\ -a + b - c + d = -1 & \dots (4) \end{cases}$$

①に②に代入すると、

$$\begin{aligned}-3 \cdot 8 + b &= -12 \\ -24 + b &= -12 \\ \therefore b &= 12 \dots (5)\end{aligned}$$

①, ⑤を③に代入すると、

$$\begin{aligned}3 \cdot 8 - 2 \cdot 12 + c &= 6 \\ 24 - 24 + c &= 6 \\ \therefore c &= 6 \dots (6)\end{aligned}$$

①, ⑤, ⑥を④に代入すると、

$$\begin{aligned}-8 + 12 - 6 + d &= -1 \\ -2 + d &= -1 \\ \therefore d &= 1\end{aligned}$$

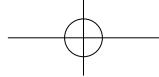
よって、

$$a = 8, b = 12, c = 6, d = 1$$

<別解1>

x についての恒等式ならば、どんな x についても等式が成り立つから、
 $x = 0$ を代入して、

$$\begin{aligned}(2 \cdot 0 - 1)^3 &= a(0 - 1)^3 + b(0 - 1)^2 + c(0 - 1) + d \\ -1 &= -a + b - c + d\end{aligned}$$



$x = 1$ を代入すると,

$$\begin{aligned}(2 \cdot 1 - 1)^3 &= a(1 - 1)^3 + b(1 - 1)^2 + c(1 - 1) + d \\ 1 &= d\end{aligned}$$

$x = 2$ を代入すると,

$$\begin{aligned}(2 \cdot 2 - 1)^3 &= a(2 - 1)^3 + b(2 - 1)^2 + c(2 - 1) + d \\ 27 &= a + b + c + d\end{aligned}$$

$x = 3$ を代入すると,

$$\begin{aligned}(2 \cdot 3 - 1)^3 &= a(3 - 1)^3 + b(3 - 1)^2 + c(3 - 1) + d \\ 125 &= 8a + 4b + 2c + d\end{aligned}$$

となる。これを整理すると,

$$\begin{cases} -a + b - c + d = -1 & \cdots ① \\ d = 1 & \cdots ② \\ a + b + c + d = 27 & \cdots ③ \\ 8a + 4b + 2c + d = 125 & \cdots ④ \end{cases}$$

②を①, ③, ④にそれぞれ代入すると,

$$\begin{cases} -a + b - c = -2 & \cdots ⑤ \\ a + b + c = 26 & \cdots ⑥ \\ 8a + 4b + 2c = 124 & \cdots ⑦ \end{cases}$$

⑤ + ⑥ より,

$$\begin{aligned}2b &= 24 \\ b &= 12\end{aligned}$$

これを⑥, ⑦にそれぞれ代入すると,

$$\begin{cases} a + c = 14 & \cdots ⑧ \\ 8a + 2c = 76 & \cdots ⑨ \end{cases}$$

⑨ ÷ 2 - ⑧ より,

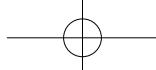
$$\begin{aligned}3a &= 24 \\ a &= 8\end{aligned}$$

これを⑧に代入して,

$$\begin{aligned}8 + c &= 14 \\ c &= 6\end{aligned}$$

を得る。逆を確かめる。

$$\begin{aligned}\text{左辺} &= (2x - 1)^3 = 8x^3 - 12x^2 + 6x - 1 \\ \text{右辺} &= 8(x - 1)^3 + 12(x - 1)^2 + 6(x - 1) + 1 \\ &= 8(x^3 - 3 \cdot x^2 \cdot 1 + 3 \cdot x \cdot 1^2 - 1^3) + 12(x^2 - 2x + 1) + 6x - 6 + 1 \\ &= 8(x^3 - 3x^2 + 3x - 1) + 12x^2 - 24x + 12 + 6x - 6 + 1 \\ &= 8x^3 - 24x^2 + 24x - 8 + 12x^2 - 24x + 12 + 6x - 6 + 1 \\ &= 8x^3 - 12x^2 + 6x - 1\end{aligned}$$



よって、左辺 = 右辺となり、確かに x についての恒等式である。よって、

$$a = 8, b = 12, c = 6, d = 1$$

<別解 2>

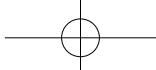
$x - 1 = k$ とおくと、 $x = k + 1$ だから、

$$\begin{aligned} \text{左辺} &= \{2(k+1) - 1\}^3 \\ &= (2k+2-1)^3 \\ &= (2k+1)^3 \\ &= (2k)^3 + 3 \cdot (2k)^2 \cdot 1 + 3 \cdot 2k \cdot 1^2 + 1^3 \\ &= 2^3 \cdot k^3 + 3 \cdot 2^2 \cdot k^2 \cdot 1 + 6k + 1 \\ &= 8k^3 + 12k^2 + 6k + 1 \\ &= 8(x-1)^3 + 12(x-1)^2 + 6(x-1) + 1 \end{aligned}$$

よって、両辺を比較すると、

$$a = 8, b = 12, c = 6, d = 1$$

- 【10】 (1)
$$\begin{aligned} (a+b-c+1)^2 &= \{(a+b)-(c-1)\}^2 \\ &= (A-B)^2 \quad [A=a+b, B=c-1 \text{ とおく}] \\ &= A^2 - 2AB + B^2 \\ &= (a+b)^2 - 2(a+b)(c-1) + (c-1)^2 \\ &= a^2 + 2ab + b^2 - (2a+2b)(c-1) + c^2 - 2c + 1 \\ &= a^2 + 2ab + b^2 - 2a(c-1) - 2b(c-1) + c^2 - 2c + 1 \\ &= a^2 + 2ab + b^2 - 2ac + 2a - 2bc + 2b + c^2 - 2c + 1 \\ &= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca + 2a + 2b - 2c + 1 \end{aligned}$$
- (2)
$$\begin{aligned} (a-b+c-1)^2 &= (A+B)^2 \quad [A=a-b, B=c-1 \text{ とおく}] \\ &= A^2 + 2AB + B^2 \\ &= (a-b)^2 + 2(a-b)(c-1) + (c-1)^2 \\ &= a^2 - 2ab + b^2 + 2a(c-1) - 2b(c-1) + c^2 - 2c + 1 \\ &= a^2 - 2ab + b^2 + 2ac - 2a - 2bc + 2b + c^2 - 2c + 1 \\ &= a^2 + b^2 + c^2 - 2ab - 2bc + 2ca - 2a + 2b - 2c + 1 \end{aligned}$$
- (3)
$$\begin{aligned} (a-b-c+d)^2 &= \{(a-b)-(c-d)\}^2 \\ &= (A-B)^2 \quad [A=a-b, B=c-d \text{ とおく}] \\ &= A^2 - 2AB + B^2 \\ &= (a-b)^2 - 2(a-b)(c-d) + (c-d)^2 \\ &= a^2 - 2ab + b^2 - (2a-2b)(c-d) + c^2 - 2cd + d^2 \\ &= a^2 - 2ab + b^2 - 2a(c-d) + 2b(c-d) + c^2 - 2cd + d^2 \\ &= a^2 - 2ab + b^2 - 2ac + 2ad + 2bc - 2bd + c^2 - 2cd + d^2 \\ &= a^2 + b^2 + c^2 + d^2 \\ &\quad - 2ab + 2bc - 2ca + 2ad - 2bd - 2cd \end{aligned}$$



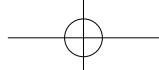
$$\begin{aligned}
 (4) \quad (a+b-c-d)^2 &= \{(a+b)-(c+d)\}^2 \\
 &= (A-B)^2 \quad [A=a+b, B=c+d \text{ とおく}] \\
 &= A^2 - 2AB + B^2 \\
 &= (a+b)^2 - 2(a+b)(c+d) + (c+d)^2 \\
 &= a^2 + 2ab + b^2 - 2\{a(c+d) + b(c+d)\} + c^2 + 2cd + d^2 \\
 &= a^2 + 2ab + b^2 - 2(ac+ad+bc+bd) + c^2 + 2cd + d^2 \\
 &= a^2 + 2ab + b^2 - 2ac - 2ad - 2bc - 2bd + c^2 + 2cd + d^2 \\
 &= \mathbf{a^2 + b^2 + c^2 + d^2} \\
 &\quad + 2ab - 2ac - 2ad - 2bc - 2bd + 2cd
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad (a^2+a+4)(a^2+a-6) &= \{(a^2+a)+4\}\{(a^2+a)-6\} \\
 &= (A+4)(A-6) \quad [A=a^2+a \text{ とおく}] \\
 &= A^2 - 2A - 24 \\
 &= (a^2+a)^2 - 2(a^2+a) - 24 \\
 &= a^4 + 2a^3 + a^2 - 2a^2 - 2a - 24 \\
 &= \mathbf{a^4 + 2a^3 - a^2 - 2a - 24}
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad (x^2+3x+2)(x^2-x+2) &= (x^2+2+3x)(x^2+2-x) \\
 &= \{(x^2+2)+3x\}\{(x^2+2)-x\} \\
 &= (A+3x)(A-x) \quad [A=x^2+2 \text{ とおく}] \\
 &= A^2 + 2xA - 3x^2 \\
 &= (x^2+2)^2 + 2x(x^2+2) - 3x^2 \\
 &= x^4 + 4x^2 + 4 + 2x^3 + 4x - 3x^2 \\
 &= \mathbf{x^4 + 2x^3 + x^2 + 4x + 4}
 \end{aligned}$$

$$\begin{aligned}
 (7) \quad (x^2-xy+2y^2)(x^2+xy+2y^2) &= (x^2+2y^2-xy)(x^2+2y^2+xy) \\
 &= \{(x^2+2y^2)-xy\}\{(x^2+2y^2)+xy\} \\
 &= (A-xy)(A+xy) \\
 &\quad [A=x^2+2y^2 \text{ とおく}] \\
 &= A^2 - (xy)^2 \\
 &= (x^2+2y^2)^2 - x^2y^2 \\
 &= x^4 + 2 \cdot x^2 \cdot 2y^2 + 4y^4 - x^2y^2 \\
 &= x^4 + 4x^2y^2 + 4y^4 - x^2y^2 \\
 &= \mathbf{x^4 + 3x^2y^2 + 4y^4}
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad (x^2+2xy+4y^2)(x^2-2xy+4y^2) &= (x^2+4y^2+2xy)(x^2+4y^2-2xy) \\
 &= \{(x^2+4y^2)+2xy\}\{(x^2+4y^2)-2xy\} \\
 &= (A+2xy)(A-2xy) \\
 &\quad [A=x^2+4y^2 \text{ とおく}] \\
 &= A^2 - (2xy)^2 \\
 &= (x^2+4y^2)^2 - 4x^2y^2 \\
 &= (x^2)^2 + 2 \cdot x^2 \cdot 4y^2 + (4y^2)^2 - 4x^2y^2 \\
 &= x^4 + 8x^2y^2 + 16y^4 - 4x^2y^2 \\
 &= \mathbf{x^4 + 4x^2y^2 + 16y^4}
 \end{aligned}$$



$$\begin{aligned}
 (9) \quad & (a-b+c-1)(a+b-c-1) = (a-1-b+c)(a-1+b-c) \\
 & = \{(a-1)-(b-c)\}\{(a-1)+(b-c)\} \\
 & = (A-B)(A+B) \\
 & \quad [A=a-1, B=b-c \text{ とおく}] \\
 & = A^2 - B^2 \\
 & = (a-1)^2 - (b-c)^2 \\
 & = a^2 - 2a + 1 - (b^2 - 2bc + c^2) \\
 & = \mathbf{a^2 - 2a + 1 - b^2 + 2bc - c^2}
 \end{aligned}$$

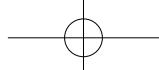
$$\begin{aligned}
 (10) \quad & (1-a+b-c)(1+c-a-b) = (1-a+b-c)(1-a-b+c) \\
 & = \{(1-a)+(b-c)\}\{(1-a)-(b-c)\} \\
 & = (A+B)(A-B) \\
 & \quad [A=1-a, B=b-c \text{ とおく}] \\
 & = A^2 - B^2 \\
 & = (1-a)^2 - (b-c)^2 \\
 & = 1 - 2a + a^2 - (b^2 - 2bc + c^2) \\
 & = \mathbf{1 - 2a + a^2 - b^2 + 2bc - c^2}
 \end{aligned}$$

$$\begin{aligned}
 (11) \quad & (1+x+x^2+x^3)(1-x+x^2-x^3) \\
 & = (1+x^2+x+x^3)(1+x^2-x-x^3) \\
 & = \{(1+x^2)+(x+x^3)\}\{(1+x^2)-(x+x^3)\} \\
 & = (A+B)(A-B) \quad [A=1+x^2, B=x+x^3 \text{ とおく}] \\
 & = A^2 - B^2 \\
 & = (1+x^2)^2 - (x+x^3)^2 \\
 & = 1 + 2x^2 + (x^2)^2 - \{x^2 + 2 \cdot x \cdot x^3 + (x^3)^2\} \\
 & = 1 + 2x^2 + x^{2 \times 2} - (x^2 + 2x^{1+3} + x^{3 \times 2}) \\
 & = 1 + 2x^2 + x^4 - (x^2 + 2x^4 + x^6) \\
 & = 1 + 2x^2 + x^4 - x^2 - 2x^4 - x^6 \\
 & = \mathbf{1 + x^2 - x^4 - x^6}
 \end{aligned}$$

$$\begin{aligned}
 (12) \quad & (8x^3 - 8x^2 + 4x - 1)(8x^3 + 8x^2 + 4x + 1) \\
 & = (8x^3 + 4x - 8x^2 - 1)(8x^3 + 4x + 8x^2 + 1) \\
 & = \{(8x^3 + 4x) - (8x^2 + 1)\}\{(8x^3 + 4x) + (8x^2 + 1)\} \\
 & = (A-B)(A+B) \quad [A=8x^3 + 4x, B=8x^2 + 1 \text{ とおく}] \\
 & = A^2 - B^2 \\
 & = (8x^3 + 4x)^2 - (8x^2 + 1)^2 \\
 & = (8x^3)^2 + 2 \cdot 8x^3 \cdot 4x + (4x)^2 - \{(8x^2)^2 + 2 \cdot 8x^2 \cdot 1 + 1^2\} \\
 & = 8^2 \cdot (x^3)^2 + 64x^{3+1} + 4^2 \cdot x^2 - \{8^2 \cdot (x^2)^2 + 16x^2 + 1\} \\
 & = 64x^{3 \times 2} + 64x^4 + 16x^2 - (64x^{2 \times 2} + 16x^2 + 1) \\
 & = 64x^6 + 64x^4 + 16x^2 - (64x^4 + 16x^2 + 1) \\
 & = 64x^6 + 64x^4 + 16x^2 - 64x^4 - 16x^2 - 1 \\
 & = \mathbf{64x^6 - 1}
 \end{aligned}$$

【11】(1) $(x^2 - 1)(x^3 + ax^2 + 3)$ において、 x^2 の係数だけ計算すると $-a + 3$ だから、

$$\begin{aligned}
 -a + 3 &= 0 \\
 a &= \mathbf{3}
 \end{aligned}$$



(2) $(x^2 + ax + b)(x^2 + bx + 2)$ において、 x^3 の係数だけ計算すると $a + b$ だから、

$$\begin{aligned}a + b &= 0 \\b &= -a \quad \cdots \textcircled{1}\end{aligned}$$

x^2 の係数だけ計算すると $2 + ab + b$ なので、

$$ab + b + 2 = 0 \quad \cdots \textcircled{2}$$

①を②に代入すると、

$$\begin{aligned}a \cdot (-a) + (-a) + 2 &= 0 \\-a^2 - a + 2 &= 0 \\a^2 + a - 2 &= 0 \\(a + 2)(a - 1) &= 0 \\a &= -2, 1\end{aligned}$$

$a = 1$ を①に代入すると、 $b = -1$

$a = -2$ を①に代入すると、 $b = 2$

よって、

$$(a, b) = (1, -1), (-2, 2)$$

(3) $(x^2 + ax + b)(x^2 - 2bx + a)$ において、 x^3 の係数だけ計算すると $a - 2b$ だから、

$$\begin{aligned}a - 2b &= 7 \\a &= 2b + 7 \quad \cdots \textcircled{1}\end{aligned}$$

x^2 の係数だけ計算すると $a - 2ab + b$ なので、

$$a - 2ab + b = 13 \quad \cdots \textcircled{2}$$

①を②に代入すると

$$\begin{aligned}(2b + 7) - 2b(2b + 7) + b &= 13 \\2b + 7 - 4b^2 - 14b + b &= 13 \\-4b^2 - 11b + 7 &= 13 \\-4b^2 - 11b - 6 &= 0 \\4b^2 + 11b + 6 &= 0 \\(b + 2)(4b + 3) &= 0 \\b &= -2, -\frac{3}{4}\end{aligned}$$

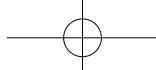
b は整数だから、 $b = -2$

これを①に代入して、

$$\begin{aligned}a &= 2 \cdot (-2) + 7 = -4 + 7 \\\therefore a &= 3\end{aligned}$$

を得る。よって、

$$a = 3, b = -2$$



(4) x^4 の係数が 1 なので、2 次式を $x^2 + mx + n$ とおくと、

$$\begin{aligned}(x^2 + mx + n)^2 &= (x^2)^2 + (mx)^2 + n^2 + 2 \cdot x^2 \cdot mx + 2 \cdot mx \cdot n + 2 \cdot n \cdot x^2 \\&= x^{2 \times 2} + m^2 x^2 + n^2 + 2mx^{2+1} + 2mn x + 2n x^2 \\&= x^4 + m^2 x^2 + n^2 + 2mx^3 + 2mn x + 2n x^2 \\&= x^4 + 2mx^3 + (m^2 + 2n)x^2 + 2mn x + n^2\end{aligned}$$

これより、

$$x^4 - 6x^3 + ax^2 + 6x + b = x^4 + 2mx^3 + (m^2 + 2n)x^2 + 2mn x + n^2$$

となる。両辺の係数を比較すると、

$$\begin{cases} -6 = 2m & \cdots ① \\ a = m^2 + 2n & \cdots ② \\ 6 = 2mn & \cdots ③ \\ b = n^2 & \cdots ④ \end{cases}$$

を得る。①より、

$$\begin{aligned}2m &= -6 \\m &= -3 \cdots ⑤\end{aligned}$$

⑤を③に代入すると、

$$\begin{aligned}2n \cdot (-3) &= 6 \\-6n &= 6 \\n &= -1 \cdots ⑥\end{aligned}$$

⑤, ⑥を②に代入して、

$$\begin{aligned}a &= (-3)^2 + 2 \cdot (-1) \\&= 9 - 2 \\&= 7\end{aligned}$$

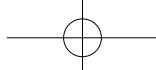
⑥を④に代入して、

$$\begin{aligned}b &= (-1)^2 \\&= 1\end{aligned}$$

よって、

$$a = 7, b = 1$$

【12】(1) 左辺 $= (x+a)(x+b)(x+c)$
 $= (x+a)(x^2 + bx + cx + bc)$
 $= x(x^2 + bx + cx + bc) + a(x^2 + bx + cx + bc)$
 $= x^3 + bx^2 + cx^2 + bc x + ax^2 + ab x + ac x + abc$
 $= x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$
 $=$ 右辺 (証明終)



$$(2) \quad \begin{aligned} & (x+1)(x+2)(x+4) \\ &= x^3 + (1+2+4)x^2 + (1\cdot 2 + 2\cdot 4 + 4\cdot 1)x + 1\cdot 2\cdot 4 \\ &= x^3 + 7x^2 + 14x + 8 \end{aligned}$$

$$\begin{aligned} & (2) \quad (a+2)(a+3)(a+4) \\ &= a^3 + (2+3+4)a^2 + (2\cdot 3 + 3\cdot 4 + 4\cdot 2)a + 2\cdot 3\cdot 4 \\ &= a^3 + 9a^2 + 26a + 24 \end{aligned}$$

【13】 (1) 左辺 $= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
 $= a(a^2+b^2+c^2-ab-bc-ca) + b(a^2+b^2+c^2-ab-bc-ca)$
 $\quad \quad \quad + c(a^2+b^2+c^2-ab-bc-ca)$
 $= a^3 + ab^2 + ac^2 - a^2b - abc - a^2c + a^2b + b^3 + bc^2 - ab^2 - b^2c - abc$
 $\quad \quad \quad + a^2c + b^2c + c^3 - abc - bc^2 - ac^2$
 $= a^3 + b^3 + c^3 - 3abc$
 $= \text{右辺} \quad (\text{証明終})$

$$(2) \quad \begin{aligned} (x+y+1)(x^2+y^2+1-xy-x-y) &= x^3 + y^3 + 1^3 - 3 \cdot x \cdot y \cdot 1 \\ &= x^3 + y^3 + 1 - 3xy \end{aligned}$$

$$\begin{aligned} & (2) \quad (a+b-1)(a^2-ab+b^2+a+b+1) \\ &= (a+b-1)(a^2+b^2+1-ab+b+a) \\ &= a^3 + b^3 + (-1)^3 - 3 \cdot a \cdot b \cdot (-1) \\ &= a^3 + b^3 - 1 + 3ab \end{aligned}$$

【14】 (1) $\begin{aligned} & \frac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\} \\ &= \frac{1}{2}(a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2) \\ &= \frac{1}{2}(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca) \\ &= a^2 + b^2 + c^2 - ab - bc - ca \end{aligned}$

$$(2) \quad \begin{aligned} (a+b)^3 - 3ab(a+b) &= a^3 + 3a^2b + 3ab^2 + b^3 - 3a^2b - 3ab^2 \\ &= a^3 + b^3 \end{aligned}$$

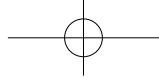
$$(3) \quad \begin{aligned} (a-b)^3 + 3ab(a-b) &= a^3 - 3a^2b + 3ab^2 - b^3 + 3a^2b - 3ab^2 \\ &= a^3 - b^3 \end{aligned}$$

(4) パスカルの三角形より,

$$(a+b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$$

$$(5) \quad \begin{aligned} (a-b)(a^3 + a^2b + ab^2 + b^3) &= a(a^3 + a^2b + ab^2 + b^3) - b(a^3 + a^2b + ab^2 + b^3) \\ &= a^4 + a^3b + a^2b^2 + ab^3 - a^3b - a^2b^2 - ab^3 - b^4 \\ &= a^4 - b^4 \end{aligned}$$

$$\begin{aligned} (6) \quad & (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4) \\ &= a(a^4 + a^3b + a^2b^2 + ab^3 + b^4) - b(a^4 + a^3b + a^2b^2 + ab^3 + b^4) \\ &= a^5 + a^4b + a^3b^2 + a^2b^3 + ab^4 - a^4b - a^3b^2 - a^2b^3 - ab^4 - b^5 \\ &= a^5 - b^5 \end{aligned}$$



$$\begin{aligned}
 (7) \quad & (a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4) \\
 & = a(a^4 - a^3b + a^2b^2 - ab^3 + b^4) + b(a^4 - a^3b + a^2b^2 - ab^3 + b^4) \\
 & = a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4 + a^4b - a^3b^2 + a^2b^3 - ab^4 + b^5 \\
 & = \mathbf{a^5 + b^5}
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad & (a+b)(a^6 - a^5b + a^4b^2 - a^3b^3 + a^2b^4 - ab^5 + b^6) \\
 & = a(a^6 - a^5b + a^4b^2 - a^3b^3 + a^2b^4 - ab^5 + b^6) \\
 & \quad + b(a^6 - a^5b + a^4b^2 - a^3b^3 + a^2b^4 - ab^5 + b^6) \\
 & = a^7 - a^6b + a^5b^2 - a^4b^3 + a^3b^4 - a^2b^5 + ab^6 \\
 & \quad + a^6b - a^5b^2 + a^4b^3 - a^3b^4 + a^2b^5 - ab^6 + b^7 \\
 & = \mathbf{a^7 + b^7}
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad & (a+b)(a^3 - a^2b + ab^2 - b^3) \\
 & = a(a^3 - a^2b + ab^2 - b^3) + b(a^3 - a^2b + ab^2 - b^3) \\
 & = a^4 - a^3b + a^2b^2 - ab^3 + a^3b - a^2b^2 + ab^3 - b^4 \\
 & = \mathbf{a^4 - b^4}
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & (a+b)(a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4 - b^5) \\
 & = a(a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4 - b^5) + b(a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4 - b^5) \\
 & = a^6 - a^5b + a^4b^2 - a^3b^3 + a^2b^4 - ab^5 + a^5b - a^4b^2 + a^3b^3 - a^2b^4 + ab^5 - b^6 \\
 & = \mathbf{a^6 - b^6}
 \end{aligned}$$

一般に次のこと成り立つ。

$$\blacklozenge (a-b)(a^{n-1} + a^{n-2}b + \cdots + ab^{n-2} + b^{n-1}) = a^n - b^n$$

$\blacklozenge n$ が奇数のとき

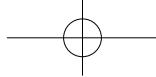
$$(a+b)(a^{n-1} - a^{n-2}b + \cdots - ab^{n-2} + b^{n-1}) = a^n + b^n$$

$\blacklozenge n$ が偶数のとき

$$(a+b)(a^{n-1} - a^{n-2}b + \cdots + ab^{n-2} - b^{n-1}) = a^n - b^n$$

$$\begin{aligned}
 [15] (1) \quad & (a+b+c)^2 - (a+b-c)^2 \\
 & = \{(a+b+c) + (a+b-c)\}\{(a+b+c) - (a+b-c)\} \\
 & = (2a+2b) \cdot 2c \\
 & = \mathbf{4ac + 4bc}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & (a+b+c)^3 - (a+b-c)^3 \\
 & = \{(a+b) + c\}^3 - \{(a+b) - c\}^3 \\
 & = (A+c)^3 - (A-c)^3 \quad [A = a+b \text{ とおく}] \\
 & = A^3 + 3A^2c + 3Ac^2 + c^3 - (A^3 - 3A^2c + 3Ac^2 - c^3) \\
 & = A^3 + 3A^2c + 3Ac^2 - A^3 + 3A^2c - 3Ac^2 + c^3 \\
 & = 6A^2c + 2c^3 \\
 & = 6c(a+b)^2 + 2c^3 \\
 & = 6c(a^2 + 2ab + b^2) + 2c^3 \\
 & = \mathbf{6a^2c + 12abc + 6b^2c + 2c^3}
 \end{aligned}$$



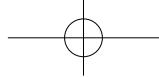
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$$\begin{aligned}
 & (a+b+c)^3 - (a+b-c)^3 \\
 &= \{(a+b)+c\}^3 - \{(a+b)-c\}^3 \\
 &= (A+c)^3 - (A-c)^3 \quad [A = a+b \text{ とおく}] \\
 &= \{(A+c) - (A-c)\} \{(A+c)^2 + (A+c)(A-c) + (A-c)^2\} \\
 &= 2c(A^2 + 2Ac + c^2 + A^2 - c^2 + A^2 - 2Ac + c^2) \\
 &= 2c(3A^2 + c^2) \\
 &= 6A^2c + 2c^3 \\
 &= 6c(a+b)^2 + 2c^3 \\
 &= 6c(a^2 + 2ab + b^2) + 2c^3 \\
 &= \mathbf{6a^2c + 12abc + 6b^2c + 2c^3}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad & a(a-b)(a-c) + b(b-c)(b-a) + c(c-a)(c-b) \\
 &= a(a^2 - ab - ac + bc) + b(b^2 - bc - ab + ca) + c(c^2 - ca - bc + ab) \\
 &= a^3 - a^2b - a^2c + abc + b^3 - b^2c - ab^2 + abc + c^3 - c^2a - c^2b + abc \\
 &= a^3 + b^3 + c^3 + 3abc - a^2b - a^2c - ab^2 - b^2c - ac^2 - bc^2
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad & (a+b+c)(a-b+c)(a+b-c)(-a+b+c) \\
 &= (a+b+c)(-a+b+c)(a-b+c)(a+b-c) \\
 &= \{a + (b+c)\} \{-a + (b+c)\} \times \{a - (b-c)\} \{a + (b-c)\} \\
 &= (a+A)(-a+A) \times (a-B)(a+B) \quad [A = b+c, B = b-c \text{ とおく}] \\
 &= (-a^2 + A^2)(a^2 - B^2) \\
 &= -(a^2 - A^2)(a^2 - B^2) \\
 &= -\{a^4 - a^2(A^2 + B^2) + A^2B^2\} \\
 &= -a^4 + a^2(A^2 + B^2) - (AB)^2 \\
 &= -a^4 + a^2\{(b+c)^2 + (b-c)^2\} - \{(b+c)(b-c)\}^2 \\
 &= -a^4 + a^2(b^2 + 2bc + c^2 + b^2 - 2bc + c^2) - (b^2 - c^2)^2 \\
 &= -a^4 + a^2(2b^2 + 2c^2) - \{(b^2)^2 - 2b^2c^2 + (c^2)^2\} \\
 &= -a^4 + 2a^2b^2 + 2a^2c^2 - (b^{2\times 2} - 2b^2c^2 + c^{2\times 2}) \\
 &= -a^4 + 2a^2b^2 + 2a^2c^2 - (b^4 - 2b^2c^2 + c^4) \\
 &= -a^4 + 2a^2b^2 + 2a^2c^2 - b^4 + 2b^2c^2 - c^4 \\
 &= \mathbf{-a^4 - b^4 - c^4 + 2a^2b^2 + 2b^2c^2 + 2c^2a^2}
 \end{aligned}$$

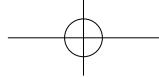
$$\begin{aligned}
 (5) \quad & (a+b+c)^2 + (-a+b+c)^2 + (a-b+c)^2 + (a+b-c)^2 \\
 &= \{a + (b+c)\}^2 + \{-a + (b+c)\}^2 + \{a - (b-c)\}^2 + \{a + (b-c)\}^2 \\
 &= (a+A)^2 + (-a+A)^2 + (a-B)^2 + (a+B)^2 \\
 & \qquad \qquad \qquad [A = b+c, B = b-c \text{ とおく}] \\
 &= a^2 + 2aA + A^2 + a^2 - 2aA + A^2 + a^2 - 2aB + B^2 + a^2 + 2aB + B^2 \\
 &= 4a^2 + 2A^2 + 2B^2 \\
 &= 4a^2 + 2(b+c)^2 + 2(b-c)^2 \\
 &= 4a^2 + 2(b^2 + 2bc + c^2) + 2(b^2 - 2bc + c^2) \\
 &= 4a^2 + 2b^2 + 4bc + 2c^2 + 2b^2 - 4bc + 2c^2 \\
 &= \mathbf{4a^2 + 4b^2 + 4c^2}
 \end{aligned}$$



$$\begin{aligned}(6) \quad & (a+b+c)^2 - (-a+b+c)^2 + (a-b+c)^2 - (a+b-c)^2 \\&= \{(a+b+c) + (-a+b+c)\} \{(a+b+c) - (-a+b+c)\} \\&\quad + \{(a-b+c) + (a+b-c)\} \{(a-b+c) - (a+b-c)\} \\&= (2b+2c) \cdot 2a + 2a(-2b+2c) \\&= 4ab + 4ac - 4ab + 4ac \\&= \mathbf{8ac}\end{aligned}$$

$$\begin{aligned}(7) \quad & (a+b+c)(a+b-c) + (a+b-c)(a-b-c) + (a-b-c)(a+b+c) \\&= \{(a+b)+c\} \{(a+b)-c\} + \{(a-c)+b\} \{(a-c)-b\} \\&\quad + \{a-(b+c)\} \{a+(b+c)\} \\&= (A+c)(A-c) + (B+b)(B-b) + (a-C)(a+C) \\&\quad [A=a+b, B=a-c, C=b+c \text{ とおく}] \\&= A^2 - c^2 + B^2 - b^2 + a^2 - C^2 \\&= (a+b)^2 - c^2 + (a-c)^2 - b^2 + a^2 - (b+c)^2 \\&= a^2 + 2ab + b^2 - c^2 + a^2 - 2ac + c^2 - b^2 + a^2 - (b^2 + 2bc + c^2) \\&= a^2 + 2ab + b^2 - c^2 + a^2 - 2ac + c^2 - b^2 + a^2 - b^2 - 2bc - c^2 \\&= \mathbf{3a^2 - b^2 - c^2 + 2ab - 2bc - 2ca}\end{aligned}$$

$$\begin{aligned}(8) \quad & (x-b)(x-c)(b-c) + (x-c)(x-a)(c-a) + (x-a)(x-b)(a-b) \\&= \{x^2 - (b+c)x + bc\} (b-c) + \{x^2 - (c+a)x + ca\} (c-a) \\&\quad + \{x^2 - (a+b)x + ab\} (a-b) \\&= (b-c)x^2 - (b+c)(b-c)x + bc(b-c) \\&\quad + (c-a)x^2 - (c+a)(c-a)x + ca(c-a) \\&\quad + (a-b)x^2 - (a+b)(a-b)x + ab(a-b) \\&= (b-c + c-a + a-b)x^2 - \{(b+c)(b-c) + (c+a)(c-a) + (a+b)(a-b)\}x \\&\quad + bc(b-c) + ca(c-a) + ab(a-b) \\&= -(b^2 - c^2 + c^2 - a^2 + a^2 - b^2)x + b^2c - bc^2 + c^2a - ca^2 + a^2b - ab^2 \\&= \mathbf{b^2c - bc^2 + c^2a - ca^2 + a^2b - ab^2}\end{aligned}$$



添削課題

[1] (1)
$$\begin{aligned}(2x - 3y)^3 &= (2x)^3 - 3 \cdot (2x)^2 \cdot 3y + 3 \cdot 2x \cdot (3y)^2 - (3y)^3 \\&= 8x^3 - 36x^2y + 54xy^2 - 27y^3\end{aligned}$$

(2)
$$(a - 3)(a^2 + 3a + 9) = a^3 - 3^3 = a^3 - 27$$

(3)
$$\begin{aligned}(2m + 3n)(4m^2 - 6mn + 9n^2) &= (2m)^3 + (3n)^3 \\&= 8m^3 + 27n^3\end{aligned}$$

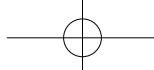
(4)
$$\begin{aligned}(3x + y - 2)^2 &= (3x)^2 + y^2 + (-2)^2 + 2 \cdot 3x \cdot y + 2 \cdot 3x \cdot (-2) + 2 \cdot y \cdot (-2) \\&= 9x^2 + 6xy + y^2 - 12x - 4y + 4\end{aligned}$$

[2] (1)
$$\begin{aligned}(2x + y)(2x + 3y)(2x - 5y) &= (2x)^3 + (y + 3y - 5y) \cdot (2x)^2 + (y \cdot 3y - 3y \cdot 5y - 5y \cdot y) \cdot 2x - y \cdot 3y \cdot 5y \\&= 8x^3 - 4x^2y - 34xy^2 - 15y^3\end{aligned}$$

(2)
$$\begin{aligned}(a + b + c)^3 - 3(b + c)(c + a)(a + b) &= \{(a + b) + c\}^3 - 3\{c^2 + (a + b)c + ab\}(a + b) \\&= (a + b)^3 + 3(a + b)^2c + 3(a + b)c^2 + c^3 - 3(a + b)c^2 - 3(a + b)^2c - 3ab(a + b) \\&= (a + b)^3 - 3ab(a + b) + c^3 \\&= a^3 + b^3 + c^3\end{aligned}$$

(3)
$$\begin{aligned}(x^8 - x^4 + 1)(x^4 - x^2 + 1)(x^2 - x + 1)(x^2 + x + 1) &= (x^8 - x^4 + 1)(x^4 - x^2 + 1)(x^4 + x^2 + 1) \\&= (x^8 - x^4 + 1)(x^8 + x^4 + 1) \\&= x^{16} + x^8 + 1\end{aligned}$$

(4)
$$\begin{aligned}(a - b + 2)(a^2 + ab + b^2 - 2a + 2b + 4) &= (a - b + 2)(a^2 + b^2 + 2^2 + ab - 2a + 2b) \\&= a^3 + (-b)^3 + 2^3 - 3 \cdot a \cdot (-b) \cdot 2 \\&= a^3 - b^3 + 6ab + 8\end{aligned}$$



【3】 x^4 の係数について

$$-6 - a + 7b = 13 \cdots ①$$

x^5 の係数について

$$-3 + 7a + 4b = -30 \cdots ②$$

①, ②より,

$$a = -5, b = 2$$

【4】 (1) 与式の左辺を k について整理すると

$$(x - 2y - 3)k + (x - 3y - 5) = 0$$

これが[§] k についての恒等式だから

$$\begin{cases} x - 2y - 3 = 0 \\ x - 3y - 5 = 0 \end{cases}$$

よって, $x = -1, y = -2$

(2) まず,

$$\begin{cases} x - y + z = 1 & \cdots ① \\ 2x + y - 2z = 0 & \cdots ② \end{cases}$$

より, y と z を x であらわすことを考える.

① + ② から,

$$3x - z = 1 \quad \therefore z = 3x - 1$$

① × 2 + ② から

$$4x - y = 2 \quad \therefore y = 4x - 2$$

これらを $ax^2 + by^2 + cz^2 = -4$ に代入して整理すると,

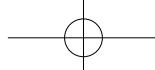
$$ax^2 + b(4x - 2)^2 + c(3x - 1)^2 = -4$$

$$(a + 16b + 9c)x^2 - 2(8b + 3c)x + (4b + c) = -4$$

この等式が x の値にかかわらず成り立つ条件は

$$\begin{cases} a + 16b + 9c = 0 \\ 8b + 3c = 0 \\ 4b + c = -4 \end{cases}$$

よって, $a = -24, b = -3, c = 8$



2章 数と式（2）－因数分解－

問題

【1】(1)

$$\begin{aligned} & x^3 + 8 \\ &= x^3 + 2^3 \\ &= (x+2)(x^2 - 2 \cdot x + 2^2) \\ &= (x+2)(x^2 - 2x + 4) \end{aligned}$$

(2)

$$\begin{aligned} & x^3y^3 + 1 \\ &= (xy)^3 + 1^3 \\ &= (xy+1)\{(xy)^2 - 1 \cdot xy + 1^2\} \\ &= (xy+1)(x^2y^2 - xy + 1) \end{aligned}$$

(3)

$$\begin{aligned} & 27a^3 - b^3 \\ &= (3a)^3 - b^3 \\ &= (3a - b)\{(3a)^2 + 3a \cdot b + b^2\} \\ &= (3a - b)(9a^2 + 3ab + b^2) \end{aligned}$$

(4)

$$\begin{aligned} & 3a^3 - 24 \\ &= 3(a^3 - 8) \\ &= 3(a^3 - 2^3) \\ &= 3\{(a-2)(a^2 + a \cdot 2 + 2^2)\} \\ &= 3(a-2)(a^2 + 2a + 4) \end{aligned}$$

(5)

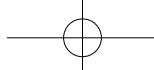
$$\begin{aligned} & a^4 + a = a(a^3 + 1) \\ &= a\{(a+1)(a^2 - a \cdot 1 + 1^2)\} \\ &= a(a+1)(a^2 - a + 1) \end{aligned}$$

(6)

$$\begin{aligned} & x^3 - (y-1)^3 = x^3 - A^3 \quad [A = y-1 \text{ とおく}] \\ &= (x-A)(x^2 + x \cdot A + A^2) \\ &= (x-A)(x^2 + xA + A^2) \\ &= \{x - (y-1)\}\{x^2 + x(y-1) + (y-1)^2\} \\ &= (x-y+1)(x^2 + xy - x + y^2 - 2y + 1) \end{aligned}$$

(7)

$$\begin{aligned} & (a-b)^4 - a + b = (a-b)^4 - (a-b) \\ &= A^4 - A \quad [A = a-b \text{ とおく}] \\ &= A(A^3 - 1) \\ &= A(A-1)(A^2 + A + 1) \\ &= (a-b)\{(a-b)-1\}\{(a-b)^2 + (a-b) + 1\} \\ &= (a-b)(a-b-1)(a^2 - 2ab + b^2 + a - b + 1) \end{aligned}$$



(8)

$$\begin{aligned} & x^3 + y^3 - z^3 + 3xyz \\ &= (x+y)^3 - 3xy(x+y) - z^3 + 3xyz \\ &= \{(x+y)^3 - z^3\} - \{3xy(x+y) - 3xyz\} \\ &= \{(x+y) - z\}\{(x+y)^2 + (x+y)z + z^2\} - 3xy(x+y-z) \\ &= (x+y-z)(x^2 + 2xy + y^2 + xz + yz + z^2) - 3xy(x+y-z) \\ &= (x+y-z)(x^2 + y^2 + z^2 - xy + yz + zx) \end{aligned}$$

(9)

$$\begin{aligned} & 8x^3 - 6xy - y^3 - 1 \\ &= (2x)^3 + (-y)^3 + (-1)^3 - 3 \cdot 2x \cdot (-y) \cdot (-1) \\ &= \{2x + (-y) + (-1)\}\{(2x)^2 + (-y)^2 + (-1)^2 - 2x \cdot (-y) \\ &\quad - (-y) \cdot (-1) - (-1) \cdot 2x\} \\ &= (2x - y - 1)(4x^2 + y^2 + 1 + 2xy - y + 2x) \end{aligned}$$

(10)

$$\begin{aligned} (a+b)^3 - a^3 - b^3 &= (a+b)^3 - (a^3 + b^3) \\ &= (a+b)^3 - (a+b)(a^2 - ab + b^2) \\ &= (a+b)\{(a+b)^2 - (a^2 - ab + b^2)\} \\ &= (a+b)(a^2 + 2ab + b^2 - a^2 + ab - b^2) \\ &= 3ab(a+b) \end{aligned}$$

【2】(1) x と y を交換すると, $12y^2 + 40yx - 12x^2$

$$\begin{aligned} & 12y^2 + 40yx - 12x^2 \neq 12x^2 + 40xy - 12y^2 \\ & 12y^2 + 40yx - 12x^2 \neq -(12x^2 + 40xy - 12y^2) \end{aligned}$$

よって, 対称式でも交代式でもない

(2) x と y を交換すると, $y^5 - x^5$

$$y^5 - x^5 = -(x^5 - y^5)$$

よって, 交代式である

(3) a と b を交換すると, $b^2 + 5ba + a^2$

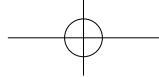
$$b^2 + 5ba + a^2 = a^2 + 5ab + b^2$$

よって, 対称式である

(4) x と y を交換すると,

$$(y+x)^2(y^2 + x^2) + 1 = (x+y)^2(x^2 + y^2) + 1$$

よって, 対称式である



(5) a と b を交換すると,

$$b^3 - 3b^2a + 3ba^2 - a^3 = -(a^3 - 3a^2b + 3ab^2 - b^3)$$

よって、交代式である

(6) a と b を交換すると、 $3b(b-a) - a(a-b)$

$$\begin{aligned} 3b(b-a) - a(a-b) &\neq 3a(a-b) - b(b-a) \\ 3b(b-a) - a(a-b) &\neq -\{3a(a-b) - b(b-a)\} \end{aligned}$$

よって、対称式でも交代式でもない

(7) ① a と b を交換すると

$$bac + ac + cb + ba + b + a + c + 1 = abc + bc + ca + ab + a + b + c + 1$$

② b と c を交換すると

$$acb + cb + ba + ac + a + c + b + 1 = abc + bc + ca + ab + a + b + c + 1$$

③ a と c を交換すると

$$cba + ba + ac + cb + c + b + a + 1 = abc + bc + ca + ab + a + b + c + 1$$

よって、 a, b, c の対称式である

(8) ① a と b を交換すると、

$$b^2(a-c) + a^2(c-b) + c^2(b-a) = -\{a^2(b-c) + b^2(c-a) + c^2(a-b)\}$$

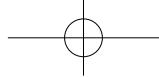
② b と c を交換すると、

$$a^2(c-b) + c^2(b-a) + b^2(a-c) = -\{a^2(b-c) + b^2(c-a) + c^2(a-b)\}$$

③ c と a を交換すると、

$$c^2(b-a) + b^2(a-c) + a^2(c-b) = -\{a^2(b-c) + b^2(c-a) + c^2(a-b)\}$$

よって、 a, b, c の交代式である



(9) ① x と y を交換すると, $y^2 + x^2 + xz - zy - 2yx$

$$\begin{aligned} y^2 + x^2 + xz - zy - 2yx &\neq x^2 + y^2 + yz - zx - 2xy \\ y^2 + x^2 + xz - zy - 2yx &\neq -\{x^2 + y^2 + yz - zx - 2xy\} \end{aligned}$$

② y と z を交換すると, $x^2 + z^2 + zy - yx - 2xz$

$$\begin{aligned} x^2 + z^2 + zy - yz - 2xz &\neq x^2 + y^2 + yz - zx - 2xy \\ x^2 + z^2 + zy - yz - 2xz &\neq -\{x^2 + y^2 + yz - zx - 2xy\} \end{aligned}$$

③ x と z を交換すると, $z^2 + y^2 + yx - xz - 2zx$

$$\begin{aligned} z^2 + y^2 + yz - xz - 2zx &\neq x^2 + y^2 + yz - zx - 2xy \\ z^2 + y^2 + yz - xz - 2zx &\neq -\{x^2 + y^2 + yz - zx - 2xy\} \end{aligned}$$

よって、対称式でも交代式でもない

(10) ① x と y を交換すると, $y(x^2 - z^2) + x(z^2 - y^2) + z(y^2 - x^2)$

$$y(x^2 - z^2) + x(z^2 - y^2) + z(y^2 - x^2) = -\{x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)\}$$

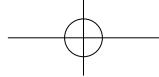
② y と z を交換すると, $x(z^2 - y^2) + z(y^2 - x^2) + y(x^2 - z^2)$

$$x(z^2 - y^2) + z(y^2 - x^2) + y(x^2 - z^2) = -\{x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)\}$$

③ z と x を交換すると, $z(y^2 - x^2) + y(x^2 - z^2) + x(z^2 - y^2)$

$$z(y^2 - x^2) + y(x^2 - z^2) + x(z^2 - y^2) = -\{x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)\}$$

よって、交代式である



(11) ① x と y を交換すると, $y^2x - y^2z + x^2z - x^2y$

$$y^2x - y^2z + x^2z - x^2y = -(x^2y - x^2z + y^2z - y^2x)$$

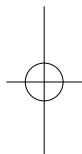
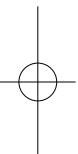
② y と z を交換すると, $x^2z - x^2y + z^2y - z^2x$

$$\begin{aligned} x^2z - x^2y + z^2y - z^2x &\neq x^2y - x^2z + y^2z - y^2x \\ x^2z - x^2y + z^2y - z^2x &\neq -(x^2y - x^2z + y^2z - y^2x) \end{aligned}$$

③ x と z を交換すると, $z^2y - z^2x + y^2x - y^2z$

$$\begin{aligned} z^2y - z^2x + y^2x - y^2z &\neq x^2y - x^2z + y^2z - y^2x \\ z^2y - z^2x + y^2x - y^2z &\neq -(x^2y - x^2z + y^2z - y^2x) \end{aligned}$$

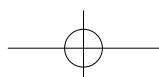
よって, 対称式でも交代式でもない

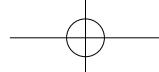


(12) a と b を交換すると, $(ba + 1)(b + 1)(a + 1) + ba$

$$(ba + 1)(b + 1)(a + 1) + ba = (ab + 1)(a + 1)(b + 1) + ab$$

よって, 対称式である





【3】(1)

$$\begin{aligned}x^2 + y^2 &= (x+y)^2 - 2xy \\&= 4^2 - 2 \cdot 1 \\&= 16 - 2 = \mathbf{14}\end{aligned}$$

(2)

$$\begin{aligned}x^3 + y^3 &= (x+y)^3 - 3xy(x+y) \\&= 4^3 - 3 \cdot 1 \cdot 4 \\&= 64 - 12 = \mathbf{52}\end{aligned}$$

<別解>

$$\begin{aligned}x^3 + y^3 &= (x+y)(x^2 - xy + y^2) \\&= (x+y)(x^2 + y^2 - xy) \\&= 4 \cdot (14 - 1) = \mathbf{52}\end{aligned}$$

(3)

$$\begin{aligned}(x-y)^2 &= (x+y)^2 - 4xy \\&= 4^2 - 4 \cdot 1 \\&= 12\end{aligned}$$

(4)

$$\begin{aligned}x^2 - y^2 &= (x+y)(x-y) \\&= 4 \cdot (\pm 2\sqrt{3}) \\&= \pm 8\sqrt{3}\end{aligned}$$

よって, $x-y = \pm 2\sqrt{3}$

(5)

$$\begin{aligned}x^4 + y^4 &= (x^2 - y^2)^2 + 2x^2y^2 \\&= (x^2 - y^2)^2 + 2(xy)^2 \\&= (\pm 8\sqrt{3})^2 + 2 \cdot 1^2 \\&= 192 + 2 \cdot 1^2 \\&= \mathbf{194}\end{aligned}$$

(6)

$$\begin{aligned}x^5 + y^5 &= (x^2 + y^2)(x^3 + y^3) - x^2y^3 - x^3y^2 \\&= (x^2 + y^2)(x^3 + y^3) - (xy)^2(x+y) \\&= 14 \cdot 52 - 1^2 \cdot 4 \\&= 728 - 4 = \mathbf{724}\end{aligned}$$

【4】(1)

$$\begin{aligned}x^2 + \frac{1}{x^2} &= \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} \\&= \left(x + \frac{1}{x}\right)^2 - 2 \\&= 4^2 - 2 = \mathbf{14}\end{aligned}$$

(2)

$$\begin{aligned}x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \\&= \left(x + \frac{1}{x}\right)^3 - 3 \left(x + \frac{1}{x}\right) \\&= 4^3 - 3 \cdot 4 = 64 - 12 = \mathbf{52}\end{aligned}$$

(3)

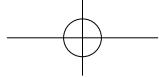
$$\begin{aligned}\left(x - \frac{1}{x}\right)^2 &= \left(x + \frac{1}{x}\right)^2 - 4 \cdot x \cdot \frac{1}{x} \quad (4) \\&= \left(x + \frac{1}{x}\right)^2 - 4 \\&= 4^2 - 4 = 16 - 4 = 12\end{aligned}$$

$$\begin{aligned}x^2 - \frac{1}{x^2} &= \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right) \\&= 4 \cdot 2\sqrt{3} = \mathbf{8\sqrt{3}}\end{aligned}$$

よって, $x - \frac{1}{x} = \pm 2\sqrt{3}$

ここで, $x > 1$ より, $x - \frac{1}{x} > 0$ な
ので,

$$x - \frac{1}{x} = 2\sqrt{3}$$



【5】(1)

$$\begin{aligned}x^2 + y^2 + z^2 &= (x + y + z)^2 - 2(xy + yz + zx) \\&= 6^2 - 2 \cdot 8 \\&= 36 - 16 = \mathbf{20}\end{aligned}$$

(2)

$$\begin{aligned}x^3 + y^3 + z^3 &= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) + 3xyz \\&= 6 \cdot (20 - 8) + 3 \cdot 5 \\&= 6 \cdot 12 + 15 = \mathbf{87}\end{aligned}$$

(3)

$$\begin{aligned}x^2y^2 + y^2z^2 + z^2x^2 &= (xy + yz + zx)^2 - 2(xy \cdot yz + yz \cdot zx + zx \cdot xy) \\&= (xy + yz + zx)^2 - 2xyz(x + y + z) \\&= 8^2 - 2 \cdot 5 \cdot 6 \\&= 64 - 60 = \mathbf{4}\end{aligned}$$

(4)

$$\begin{aligned}x^4 + y^4 + z^4 &= (x^2 + y^2 + z^2)^2 - 2(x^2y^2 + y^2z^2 + z^2x^2) \\&= 20^2 - 2 \cdot 4 \\&= 400 - 8 = \mathbf{392}\end{aligned}$$

(5)

$$\begin{aligned}\frac{1}{x} + \frac{1}{y} + \frac{1}{z} &= \frac{yz + zx + xy}{xyz} \\&= \frac{8}{5}\end{aligned}$$

(6)

$$\begin{aligned}\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} &= \frac{y^2z^2 + z^2x^2 + x^2y^2}{x^2y^2z^2} \\&= \frac{4}{5^2} = \frac{4}{\mathbf{25}}\end{aligned}$$

【6】(1) $(x+y+z)^2 = (x^2 + y^2 + z^2) + 2(xy + yz + zx)$ より,

$$\begin{aligned}3^2 &= 5 + 2(xy + yz + zx) \\2(xy + yz + zx) &= 9 - 5 \\2(xy + yz + zx) &= 4 \\xy + yz + zx &= 2\end{aligned}$$

(2) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$ より,

$$\begin{aligned}15 - 3xyz &= 3 \cdot (5 - 2) \\3xyz &= 15 - 9 = 6 \\xyz &= 2\end{aligned}$$

(3) $x+y+z = 3$ より,

$$\begin{aligned}(3-x)(3-y)(3-z) &= (3-x)(9-3y-3z+yz) \\&= 27 - 9y - 9z + 3yz - 9x + 3xy + 3xz - xyz \\&= 27 - 9(x+y+z) + 3(xy+yz+zx) - xyz \\&= 27 - 9 \cdot 3 + 3 \cdot 2 - 2 \\&= 27 - 27 + 6 - 2 \\&= 4\end{aligned}$$

【7】(1)

$$\begin{aligned} & xy + x - 3y - bx + 2ay + 2a + 3b - 2ab - 3 \\ & = x(y - b + 1) - 3(y - b + 1) + 2a(y - b + 1) \\ & = (x + 2a - 3)(y - b + 1) \end{aligned}$$

(2)

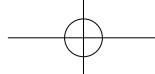
$$\begin{aligned} & x^2y - xy^2 - x^2z - xz^2 - y^2z + yz^2 + 2xyz \\ & = (y - z)x^2 - (y^2 - 2yz + z^2)x - y^2z + yz^2 \\ & = (y - z)x^2 - (y - z)^2x - yz(y - z) \\ & = (y - z)\{x^2 - (y - z)x - yz\} \\ & = (y - z)\{(x - y)(x + z)\} \\ & = (x - y)(y - z)(x + z) \end{aligned}$$

(3)

$$\begin{aligned} & (xy + 1)(x + 1)(y + 1) + xy \\ & = (xy + 1)(xy + x + y + 1) + xy \\ & = x^2y^2 + x^2y + xy^2 + xy + xy + x + y + 1 + xy \\ & = x^2y^2 + x^2y + xy^2 + 3xy + x + y + 1 \\ & = y(y + 1)x^2 + (y^2 + 3y + 1)x + (y + 1) \\ & = \{yx + (y + 1)\}\{(y + 1)x + 1\} \\ & = (xy + y + 1)(xy + x + 1) \end{aligned}$$

(4)

$$\begin{aligned} & (x + y)(y + z)(z + x) + xyz \\ & = (xy + xz + y^2 + yz)(z + x) + xyz \\ & = xyz + x^2y + xz^2 + x^2z + y^2z + y^2x + yz^2 + xyz + xyz \\ & = (y + z)x^2 + (y^2 + 3yz + z^2)x + y^2z + yz^2 \\ & = (y + z)x^2 + (y^2 + 3yz + z^2)x + yz(y + z) \\ & = \{(y + z)x + yz\}\{x + (y + z)\} \\ & = (yz + zx + xy)(x + y + z) \end{aligned}$$



【8】(1) $(x+y+z)^3 - x^3 - y^3 - z^3$ は 3 変数の対称式なので、 $x+y$ が因数ならば $y+z$, $z+x$ も因数にもつ。

$$\begin{aligned}
 & (x+y+z)^3 - x^3 - y^3 - z^3 \\
 &= x^3 + x^2y + x^2z + y^2x + y^3 + y^2z + z^2x + z^2y + z^3 \\
 &\quad + 2x^2y + 2xy^2 + 2xyz + 2xyz + 2y^2z + 2yz^2 + 2zx^2 \\
 &\quad + 2xyz + 2z^2x - x^3 - y^3 - z^3 \\
 &= 3x^2y + 3x^2z + 3xy^2 + 3xz^2 + 3y^2z + 3yz^2 + 6xyz \\
 &= 3(y+z)x^2 + 3(y^2 + z^2 + 2yz)x + 3yz(y+z) \\
 &= 3(y+z)x^2 + 3(y+z)^2x + 3yz(y+z) \\
 &= 3(y+z)\{x^2 + (y+z)x + yz\} \\
 &= 3(y+z)(x+y)(x+z)
 \end{aligned}$$

(2)

$$\begin{aligned}
 & (a+b+c)(ab+bc+ca) - abc \\
 &= a^2b + abc + a^2c + ab^2 + b^2c + abc + abc + bc^2 + ac^2 - abc \\
 &= (b+c)a^2 + (2bc + b^2 + c^2)a + (b^2c + bc^2) \\
 &= (b+c)a^2 + (b+c)^2a + bc(b+c)
 \end{aligned}$$

3 変数の対称式なので、 $b+c$ が因数ならば $a+b$, $c+a$ も因数にもつ。よって、

$$\begin{aligned}
 \text{与式} &= (b+c)\{a^2 + (b+c)a + bc\} \\
 &= (b+c)(a+b)(a+c)
 \end{aligned}$$

(3)

$$\begin{aligned}
 & (x+y)z^2 + (y+z)x^2 + (z+x)y^2 + 2xyz \\
 &= xz^2 + yz^2 + yx^2 + zx^2 + zy^2 + xy^2 + 2xyz \\
 &= (y+z)x^2 + (z^2 + y^2 + 2yz)x + yz^2 + y^2z \\
 &= (y+z)x^2 + (y+z)^2x + yz(y+z)
 \end{aligned}$$

3 変数の対称式なので、 $y+z$ が因数ならば $x+y$, $z+x$ も因数にもつ。よって、

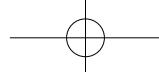
$$\begin{aligned}
 \text{与式} &= (y+z)\{x^2 + (y+z)x + yz\} \\
 &= (y+z)(x+y)(x+z)
 \end{aligned}$$

(4)

$$\begin{aligned} & a(b+c)^2 + b(c+a)^2 + c(a+b)^2 - 4abc \\ & = a(b^2 + 2bc + c^2) + b(c^2 + 2ca + a^2) + c(a^2 + 2ab + b^2) - 4abc \\ & = ab^2 + 2abc + ac^2 + bc^2 + 2abc + ba^2 + ca^2 + 2abc + cb^2 - 4abc \\ & = (b+c)a^2 + (b^2 + c^2 + 2bc)a + b^2c + bc^2 \\ & = (b+c)a^2 + (b+c)^2a + bc(b+c) \end{aligned}$$

3変数の対称式なので、 $b+c$ が因数ならば、 $a+b$, $c+a$ も因数にもつ。よって、

$$\begin{aligned} \text{与式} &= (b+c)\{a^2 + (b+c)a + bc\} \\ &= (b+c)(a+b)(a+c) \end{aligned}$$



【9】

(1)

$$\begin{aligned} xy(x-y) + yz(y-z) + zx(z-x) &= x^2y - xy^2 + y^2z - yz^2 + z^2x - zx^2 \\ &= x^2(y-z) - x(y^2 - z^2) + y^2z - yz^2 \\ &= x^2(y-z) - (y+z)(y-z)x + yz(y-z) \end{aligned}$$

交代式なので、 $(x-y)(y-z)(z-x)$ を因数にもつ。

$$\begin{aligned} \text{与式} &= (y-z)\{x^2 - (y+z)x + yz\} \\ &= (y-z)(x-z)(x-y) \\ &= -(x-y)(y-z)(z-x) \end{aligned}$$

(2)

$$\begin{aligned} a^2(b-c) + b^2(c-a) + c^2(a-b) &= a^2b - a^2c + b^2c - b^2a + c^2a - c^2b \\ &= (b-c)a^2 - (b^2 - c^2)a + bc(b-c) \end{aligned}$$

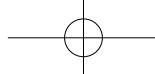
交代式なので、 $(b-c)(c-a)(a-b)$ を因数にもつ。

$$\begin{aligned} \text{与式} &= (b-c)\{a^2 - (b+c)a + bc\} \\ &= (b-c)(a-b)(a-c) \\ &= -(a-b)(b-c)(c-a) \end{aligned}$$

(3)

$$\begin{aligned} &x(y^3 - z^3) + y(z^3 - x^3) + z(x^3 - y^3) \\ &= xy^3 - xz^3 + yz^3 - yx^3 + zx^3 - zy^3 \\ &= x(y^3 - z^3) - x^3(y-z) - yz(y^2 - z^2) \\ &= x(y-z)(y^2 + yz + z^2) - x^3(y-z) - yz(y+z)(y-z) \\ &= (y-z)\{x(y^2 + yz + z^2) - x^3 - yz(y+z)\} \\ &= (y-z)\{xy^2 + xyz + xz^2 - x^3 - y^2z - yz^2\} \\ &= (y-z)\{(x-y)z^2 + (xy - y^2)z + xy^2 - x^3\} \\ &= (y-z)\{(x-y)z^2 + (xy - y^2)z - x(x^2 - y^2)\} \\ &= (y-z)\{(x-y)z^2 + (xy - y^2)z - x(x+y)(x-y)\} \\ &= (y-z)(x-y)\{z^2 + yz - x^2 - xy\} \\ &= (y-z)(x-y)\{z^2 - x^2 + y(z-x)\} \\ &= (y-z)(x-y)\{(z+x)(z-x) + y(z-x)\} \\ &= (y-z)(x-y)(z-x)(x+y+z) \end{aligned}$$

$$\begin{aligned}
(4) \quad & a^3(b - c) + b^3(c - a) + c^3(a - b) \\
& = a^3b - a^3c + b^3c - b^3a + c^3a - c^3b \\
& = a^3(b - c) + bc(b^2 - c^2) - a(b^3 - c^3) \\
& = a^3(b - c) + bc(b + c)(b - c) - a(b - c)(b^2 + bc + c^2) \\
& = (b - c)\{a^3 + bc(b + c) - a(b^2 + bc + c^2)\} \\
& = (b - c)\{a^3 + b^2c + bc^2 - ab^2 - abc - ac^2\} \\
& = (b - c)\{(b - a)c^2 + (b^2 - ab)c + a^3 - ab^2\} \\
& = (b - c)\{(b - a)c^2 + b(b - a)c - a(b + a)(b - a)\} \\
& = (b - c)(b - a)\{c^2 + bc - a(b + a)\} \\
& = (b - c)(b - a)(c^2 + bc - ab - a^2) \\
& = (b - c)(b - a)\{(c - a)b + (c + a)(c - a)\} \\
& = (b - c)(b - a)(c - a)(a + b + c) \\
& = -(a - b)(b - c)(c - a)(a + b + c)
\end{aligned}$$



添削課題

【1】(1) $x^3 + 125 = x^3 + 5^3 = (x + 5)(x^2 - 5x + 25)$

(2) $4x^5 - 32x^2 = 4x^2(x^3 - 8) = 4x^2(x - 2)(x^2 + 2x + 4)$

(3) $a^6 + 26a^3 - 27 = (a^3 - 1)(a^3 + 27)$
 $= (a - 1)(a^2 + a + 1)(a + 3)(a^2 - 3a + 9)$

(4) $m^6 - n^6 = (m^3)^2 - (n^3)^2$
 $= (m^3 - n^3)(m^3 + n^3)$
 $= (m - n)(m^2 + mn + n^2)(m + n)(m^2 - mn + n^2)$

<別解>

$$\begin{aligned} m^6 - n^6 &= (m^2)^3 - (n^2)^3 \\ &= (m^2 - n^2)(m^4 + m^2n^2 + n^4) \\ &= (m^2 - n^2) \{(m^2 + n^2)^2 - (mn)^2\} \\ &= (m - n)(m + n)(m^2 - mn + n^2)(m^2 + mn + n^2) \end{aligned}$$

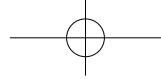
【2】(1) $a^3 + b^3 + 1 - 3ab = a^3 + b^3 + 1^3 - 3 \cdot a \cdot b \cdot 1$
 $= (a + b + 1)(a^2 + b^2 + 1 - ab - a - b)$

(2) $27a^3 - 36ab - b^3 - 64 = (3a)^3 + (-b)^3 + (-4)^3 - 3 \cdot 3a \cdot (-b) \cdot (-4)$
 $= (3a - b - 4)(9a^2 + b^2 + 16 + 3ab + 12a - 4b)$

(3) $a - x = A, b - x = B$ とおくと, $a + b - 2x = A + B$ だから,

$$\begin{aligned} \text{与式} &= A^3 + B^3 - (A + B)^3 \\ &= A^3 + B^3 - (A^3 + 3A^2B + 3AB^2 + B^3) \\ &= -3A^2B - 3AB^2 \\ &= -3AB(A + B) \\ &= -3(a - x)(b - x)(a + b - 2x) \end{aligned}$$

(4) $x^3 - 6x^2 + 12x - 8 = x^3 - 3 \cdot x^2 \cdot 2 + 3 \cdot x \cdot 2^2 - 2^3$
 $= (x - 2)^3$

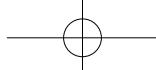


【3】 (1) $x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$
 $= 3^2 + 2 = \mathbf{11}$

(2) $x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right) \left(x^2 + 1 + \frac{1}{x^2}\right)$ (3) $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2} = 13$ たゞ
 $= 3 \cdot (11 + 1) = \mathbf{36}$ から,
 $x + \frac{1}{x} = \pm\sqrt{13}$

(4) $x^2 - \frac{1}{x^2} = \left(x - \frac{1}{x}\right) \left(x + \frac{1}{x}\right)$
 $= 3 \cdot (\pm\sqrt{13}) = \pm\mathbf{3}\sqrt{13}$

(5) $x^5 - \frac{1}{x^5} = \left(x^2 + \frac{1}{x^2}\right) \left(x^3 - \frac{1}{x^3}\right) - \left(x - \frac{1}{x}\right)$
 $= 11 \cdot 36 - 3 = \mathbf{393}$



【4】

$$\begin{cases} x + y + z = 1 & \cdots ① \\ x^2 + y^2 + z^2 = 3 & \cdots ② \\ xyz = -1 & \cdots ③ \end{cases}$$

$$(1) \quad (x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx) \text{ に } ①, ② \text{ を代入して}$$

$$1 = 3 + 2(xy + yz + zx)$$

$$\therefore xy + yz + zx = \frac{1-3}{2} = -1$$

$$(2) \quad x^3 + y^3 + z^3 = (x + y + z)\{x^2 + y^2 + z^2 - (xy + yz + zx)\} + 3xyz$$

$$= 1 \cdot (3 + 1) + 3 \cdot (-1)$$

$$= 1$$

$$(3) \quad (x + y)(y + z)(z + x) = (1 - z)(1 - x)(1 - y)$$

$$= 1 - (x + y + z) + (xy + yz + zx) - xyz$$

$$= 1 - 1 - 1 + 1 = 0$$

$$(4) \quad x^4 + y^4 + z^4 = (x^2 + y^2 + z^2)^2 - 2(x^2y^2 + y^2z^2 + z^2x^2)$$

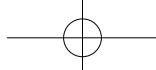
ここで、

$$x^2y^2 + y^2z^2 + z^2x^2 = (xy + yz + zx)^2 - 2xyz(x + y + z)$$

$$= (-1)^2 - 2 \cdot (-1) \cdot 1 = 3$$

だから、

$$x^4 + y^4 + z^4 = 3^2 - 2 \cdot 3 = 3$$



3章 数と式 (3) －実数－

問題

[1] (1) $|2.71| = 2.71$ (2) $|2 - 9| = |-7| = 7$

$$(3) \quad | -3.5 | - 3 | -1.5 | = 3.5 - 3 \times 1.5 \quad (4) \quad \left| -\frac{1}{3} \right| + \left| -\frac{1}{2} \right| = \frac{1}{3} + \frac{1}{2}$$

$$= 3.5 - 4.5 \\ = -1 \qquad \qquad \qquad = \frac{5}{6}$$

(5) $3 - \sqrt{10} < 0$ なので, (6) $3 - \pi < 0$ なので,

$$|3 - \sqrt{10}| = -(3 - \sqrt{10}) \quad |3 - \pi| = -(3 - \pi)$$

$$= \sqrt{10} - 3 \quad = \pi - 3$$

[2] (1) (i) $-x \geq 0$ のとき,
つまり, $x \leq 0$ のとき,
 $| -x | = -x$
(ii) $-x < 0$ のとき,
つまり, $x > 0$ のとき,
 $| -x | = -(-x) = x$

$$(i), (ii) より$$

$$| -x | = \begin{cases} -x & (x \leq 0 \text{ のとき}) \\ x & (x > 0 \text{ のとき}) \end{cases}$$

(2) (i) $7 - x \geq 0$ のとき,
つまり, $x \leq 7$ のとき,
 $| 7 - x | = 7 - x$
(ii) $7 - x < 0$ のとき,
つまり, $x > 7$ のとき
 $| 7 - x | = -(7 - x) = -7 + x$

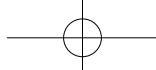
$$(i), (ii) より$$

$$| 7 - x | = \begin{cases} 7 - x & (x \leq 7 \text{ のとき}) \\ x - 7 & (x > 7 \text{ のとき}) \end{cases}$$

(3) (i) $2 + x \geq 0$ のとき,
つまり, $x \geq -2$ のとき,
 $x + | 2 + x | = x + (2 + x) = 2 + 2x$
(ii) $2 + x < 0$ のとき,
つまり, $x < -2$ のとき
 $x + | 2 + x | = x - (2 + x) = -2$

$$(i), (ii) より$$

$$x + | 2 + x | = \begin{cases} 2 + 2x & (x \geq -2 \text{ のとき}) \\ -2 & (x < -2 \text{ のとき}) \end{cases}$$



$$(4) |x| = \begin{cases} x & (x \geq 0 \text{ のとき}) \\ -x & (x < 0 \text{ のとき}) \end{cases}$$
$$|x+3| = \begin{cases} x+3 & (x+3 \geq 0, \text{ すなわち, } x \geq -3 \text{ のとき}) \\ -x-3 & (x+3 < 0, \text{ すなわち, } x < -3 \text{ のとき}) \end{cases}$$

よって、

$$(i) x < -3 \text{ のとき,} \quad (ii) -3 \leq x < 0 \text{ のとき,}$$

$$\begin{aligned} |x| - |x+3| &= -x + (x+3) & |x| - |x+3| &= -x - (x+3) \\ &= -x + x + 3 & &= -x - x - 3 \\ &= 3 & &= -2x - 3 \end{aligned}$$

$$(iii) x \geq 0 \text{ のとき,}$$

$$\begin{aligned} |x| - |x+3| &= x - (x+3) \\ &= x - x - 3 \\ &= -3 \end{aligned}$$

(i)~(iii) より、

$$|x| - |x+3| = \begin{cases} 3 & (x < -3 \text{ のとき}) \\ -2x - 3 & (-3 \leq x < 0 \text{ のとき}) \\ -3 & (x \geq 0 \text{ のとき}) \end{cases}$$

$$(5) |x-2| = \begin{cases} x-2 & (x-2 \geq 0, \text{ すなわち, } x \geq 2 \text{ のとき}) \\ -x+2 & (x-2 < 0, \text{ すなわち, } x < 2 \text{ のとき}) \end{cases}$$

$$|x+1| = \begin{cases} x+1 & (x+1 \geq 0, \text{ すなわち, } x \geq -1 \text{ のとき}) \\ -x-1 & (x+1 < 0, \text{ すなわち, } x < -1 \text{ のとき}) \end{cases}$$

$$(i) x < -1 \text{ のとき,}$$

$$\begin{aligned} |x-2| - 5|x+1| &= -(x-2) + 5(x+1) \\ &= -x + 2 + 5x + 5 \\ &= 4x + 7 \end{aligned}$$

$$(ii) -1 \leq x < 2 \text{ のとき,}$$

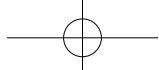
$$\begin{aligned} |x-2| - 5|x+1| &= -(x-2) - 5(x+1) \\ &= -x + 2 - 5x - 5 \\ &= -6x - 3 \end{aligned}$$

$$(iii) x \geq 2 \text{ のとき,}$$

$$\begin{aligned} |x-2| - 5|x+1| &= (x-2) - 5(x+1) \\ &= x - 2 - 5x - 5 \end{aligned}$$

(i)~(iii) より、

$$|x-2| - 5|x+1| = \begin{cases} 4x + 7 & (x < -1 \text{ のとき}) \\ -6x - 3 & (-1 \leq x < 2 \text{ のとき}) \\ -4x - 7 & (x \geq 2 \text{ のとき}) \end{cases}$$



$$[3] (1) \quad \sqrt{(\sqrt{5}-2)^2} = |\sqrt{5}-2|$$

$2 < \sqrt{5} < 3$ より, $\sqrt{5}-2 > 0$ だから,

$$\begin{aligned}\sqrt{(\sqrt{5}-2)^2} &= |\sqrt{5}-2| \\ &= \sqrt{5}-2\end{aligned}$$

$$(2) \quad \sqrt{(2\pi-7)^2} - \sqrt{(\pi+1)^2} = |2\pi-7| - |\pi+1|$$

$6 < 2\pi < 7$ より, $2\pi-7 < 0$ だから,

$$\begin{aligned}\sqrt{(2\pi-7)^2} - \sqrt{(\pi+1)^2} &= |2\pi-7| - |\pi+1| \\ &= -(2\pi-7) - (\pi+1) \\ &= -2\pi + 7 - \pi - 1 \\ &= -3\pi + 6\end{aligned}$$

$$(3) \quad \sqrt{(2x-1)^2} = |2x-1|$$

(i) $2x-1 \geq 0$, すなわち $x \geq \frac{1}{2}$ のとき,

$$\begin{aligned}\sqrt{(2x-1)^2} &= |2x-1| \\ &= 2x-1\end{aligned}$$

(ii) $2x-1 < 0$, すなわち $x < \frac{1}{2}$ のとき,

$$\begin{aligned}\sqrt{(2x-1)^2} &= |2x-1| \\ &= -(2x-1) \\ &= -2x+1\end{aligned}$$

(i), (ii) より,

$$\sqrt{(2x-1)^2} = \begin{cases} 2x-1 & \left(x \geq \frac{1}{2} \text{ のとき} \right) \\ -2x+1 & \left(x < \frac{1}{2} \text{ のとき} \right) \end{cases}$$

$$(4) \quad \sqrt{9x^2 - 6x + 1} = \sqrt{(3x-1)^2} = |3x-1|$$

(i) $3x-1 \geq 0$, すなわち $x \geq \frac{1}{3}$ のとき,

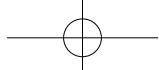
$$\begin{aligned}\sqrt{9x^2 - 6x + 1} &= |3x-1| \\ &= 3x-1\end{aligned}$$

(ii) $3x-1 < 0$, すなわち $x < \frac{1}{3}$ のとき,

$$\begin{aligned}\sqrt{9x^2 - 6x + 1} &= |3x-1| \\ &= -(3x-1) \\ &= -3x+1\end{aligned}$$

(i), (ii) より,

$$\sqrt{9x^2 - 6x + 1} = \begin{cases} 3x-1 & \left(x \geq \frac{1}{3} \text{ のとき} \right) \\ -3x+1 & \left(x < \frac{1}{3} \text{ のとき} \right) \end{cases}$$



$$(5) \quad \sqrt{(x-3)^2} - \sqrt{(x+3)^2} = |x-3| - |x+3|$$

であり,

$$|x-3| = \begin{cases} x-3 & (x-3 \geq 0, \text{ すなわち, } x \geq 3 \text{ のとき}) \\ -x+3 & (x-3 < 0, \text{ すなわち, } x < 3 \text{ のとき}) \end{cases}$$

$$|x+3| = \begin{cases} x+3 & (x+3 \geq 0, \text{ すなわち, } x \geq -3 \text{ のとき}) \\ -x-3 & (x+3 < 0, \text{ すなわち, } x < -3 \text{ のとき}) \end{cases}$$

(i) $x < -3$ のとき,

$$\begin{aligned} \sqrt{(x-3)^2} - \sqrt{(x+3)^2} &= |x-3| - |x+3| \\ &= -(x-3) + (x+3) \\ &= -x+3+x+3 \\ &= \mathbf{6} \end{aligned}$$

(ii) $-3 \leq x < 3$ のとき,

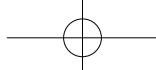
$$\begin{aligned} \sqrt{(x-3)^2} - \sqrt{(x+3)^2} &= |x-3| - |x+3| \\ &= -(x-3) - (x+3) \\ &= -x+3-x-3 \\ &= \mathbf{-2x} \end{aligned}$$

(iii) $x \geq 3$ のとき,

$$\begin{aligned} \sqrt{(x-3)^2} - \sqrt{(x+3)^2} &= |x-3| - |x+3| \\ &= (x-3) - (x+3) \\ &= x-3-x-3 \\ &= \mathbf{-6} \end{aligned}$$

(i)~(iii) より,

$$\sqrt{(x-3)^2} - \sqrt{(x+3)^2} = \begin{cases} \mathbf{6} & (x < -3 \text{ のとき}) \\ -2x & (-3 \leq x < 3 \text{ のとき}) \\ -6 & (x \geq 3 \text{ のとき}) \end{cases}$$



$$(6) \quad \sqrt{x^2 - 10x + 25} - \sqrt{4x^2 - 4x + 1} = \sqrt{(x-5)^2} - \sqrt{(2x-1)^2} \\ = |x-5| - |2x-1|$$

であり、

$$|x-5| = \begin{cases} x-5 & (x-5 \geq 0, \text{ すなわち, } x \geq 5 \text{ のとき}) \\ -x+5 & (x-5 < 0, \text{ すなわち, } x < 5 \text{ のとき}) \end{cases}$$

$$|2x-1| = \begin{cases} 2x-1 & \left(2x-1 \geq 0, \text{ すなわち, } x \geq \frac{1}{2} \text{ のとき}\right) \\ -2x+1 & \left(2x-1 < 0, \text{ すなわち, } x < \frac{1}{2} \text{ のとき}\right) \end{cases}$$

(i) $x < \frac{1}{2}$ のとき、

$$\sqrt{x^2 - 10x + 25} - \sqrt{4x^2 - 4x + 1} = |x-5| - |2x-1| \\ = -(x-5) + (2x-1) \\ = -x + 5 + 2x - 1 \\ = x + 4$$

(ii) $\frac{1}{2} \leq x < 5$ のとき、

$$\sqrt{x^2 - 10x + 25} - \sqrt{4x^2 - 4x + 1} = |x-5| - |2x-1| \\ = -(x-5) - (2x-1) \\ = -x + 5 - 2x + 1 \\ = -3x + 6$$

(iii) $x \geq 5$ のとき、

$$\sqrt{x^2 - 10x + 25} - \sqrt{4x^2 - 4x + 1} = |x-5| - |2x-1| \\ = (x-5) - (2x-1) \\ = x - 5 - 2x + 1 \\ = -x - 4$$

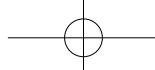
(i)～(iii) より、

$$\sqrt{x^2 - 10x + 25} - \sqrt{4x^2 - 4x + 1} = \begin{cases} x+4 & \left(x < \frac{1}{2} \text{ のとき}\right) \\ -3x+6 & \left(\frac{1}{2} \leq x < 5 \text{ のとき}\right) \\ -x-4 & \left(x \geq 5 \text{ のとき}\right) \end{cases}$$

【4】 (1) $(\sqrt{3} + \sqrt{5})^3 = (\sqrt{3})^3 + 3 \cdot (\sqrt{3})^2 \cdot \sqrt{5} + 3 \cdot \sqrt{3} \cdot (\sqrt{5})^2 + (\sqrt{5})^3$
 $= 3\sqrt{3} + 9\sqrt{5} + 15\sqrt{3} + 5\sqrt{5}$
 $= 18\sqrt{3} + 14\sqrt{5}$

(2) $(\sqrt{3} - 1)^3 = (\sqrt{3})^3 - 3 \cdot (\sqrt{3})^2 \cdot 1 + 3 \cdot \sqrt{3} \cdot 1^2 - 1^3$
 $= 3\sqrt{3} - 9 + 3\sqrt{3} - 1$
 $= 6\sqrt{3} - 10$

(3) $(\sqrt{2} + 2)^3 = (\sqrt{2})^3 + 3 \cdot (\sqrt{2})^2 \cdot 2 + 3 \cdot \sqrt{2} \cdot 2^2 + 2^3$
 $= 2\sqrt{2} + 12 + 12\sqrt{2} + 8$
 $= 14\sqrt{2} + 20$



$$\begin{aligned}(4) \quad (3\sqrt{2} + 1)^3 &= (3\sqrt{2})^3 + 3 \cdot (3\sqrt{2})^2 \cdot 1 + 3 \cdot 3\sqrt{2} \cdot 1^2 + 1^3 \\&= 54\sqrt{2} + 54 + 9\sqrt{2} + 1 \\&= \mathbf{63\sqrt{2} + 55}\end{aligned}$$

$$\begin{aligned}(5) \quad (\sqrt{2} + \sqrt{3} + \sqrt{5})^2 &= (\sqrt{2})^2 + (\sqrt{3})^2 + (\sqrt{5})^2 + 2 \cdot \sqrt{2} \cdot \sqrt{3} + 2 \cdot \sqrt{3} \cdot \sqrt{5} + 2 \cdot \sqrt{5} \cdot \sqrt{2} \\&= 2 + 3 + 5 + 2\sqrt{6} + 2\sqrt{15} + 2\sqrt{10} \\&= \mathbf{10 + 2\sqrt{6} + 2\sqrt{15} + 2\sqrt{10}}\end{aligned}$$

$$\begin{aligned}(6) \quad (\sqrt{2} - \sqrt{3} + \sqrt{5})^2 &= (\sqrt{2})^2 + (-\sqrt{3})^2 + (\sqrt{5})^2 + 2 \cdot \sqrt{2} \cdot (-\sqrt{3}) + 2 \cdot (-\sqrt{3}) \cdot \sqrt{5} + 2 \cdot \sqrt{5} \cdot \sqrt{2} \\&= 2 + 3 + 5 - 2\sqrt{6} - 2\sqrt{15} + 2\sqrt{10} \\&= \mathbf{10 - 2\sqrt{6} - 2\sqrt{15} + 2\sqrt{10}}\end{aligned}$$

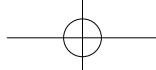
【5】 (1) $\sqrt{11 + 2\sqrt{30}} = \sqrt{(6 + 5) + 2\sqrt{6 \cdot 5}}$
 $= \sqrt{(\sqrt{6} + \sqrt{5})^2}$
 $= |\sqrt{6} + \sqrt{5}|$
 $= \sqrt{6} + \sqrt{5}$

(2) $\sqrt{7 - 2\sqrt{12}}$ (3) $\sqrt{11 + 4\sqrt{7}}$
 $= \sqrt{(4 + 3) - 2\sqrt{4 \cdot 3}}$
 $= \sqrt{(\sqrt{4} - \sqrt{3})^2}$
 $= |\sqrt{4} - \sqrt{3}|$
 $= \sqrt{4} - \sqrt{3}$
 $= \mathbf{2 - \sqrt{3}}$

(3) $\sqrt{11 + 2\sqrt{28}}$
 $= \sqrt{(7 + 4) + 2\sqrt{7 \cdot 4}}$
 $= \sqrt{(\sqrt{7} + \sqrt{4})^2}$
 $= |\sqrt{7} + \sqrt{4}|$
 $= \mathbf{\sqrt{7} + 2}$

(4) $\sqrt{3 + \sqrt{8}}$ (5) $\sqrt{17 + 3\sqrt{32}}$
 $= \sqrt{3 + 2\sqrt{2}}$
 $= \sqrt{(2 + 1) + 2\sqrt{2 \cdot 1}}$
 $= \sqrt{(\sqrt{2} + \sqrt{1})^2}$
 $= |\sqrt{2} + \sqrt{1}|$
 $= \mathbf{\sqrt{2} + 1}$

(5) $\sqrt{17 + \sqrt{288}}$
 $= \sqrt{17 + 2\sqrt{72}}$
 $= \sqrt{(9 + 8) + 2\sqrt{9 \cdot 8}}$
 $= \sqrt{(\sqrt{9} + \sqrt{8})^2}$
 $= |\sqrt{9} + \sqrt{8}|$
 $= \mathbf{3 + 2\sqrt{2}}$



$$\begin{aligned}
 (6) \quad \sqrt{3+\sqrt{5}} &= \sqrt{\frac{6+2\sqrt{5}}{2}} \\
 &= \sqrt{\frac{(5+1)+2\sqrt{5}\cdot 1}{2}} \\
 &= \sqrt{\frac{(\sqrt{5}+\sqrt{1})^2}{2}} \\
 &= \frac{|\sqrt{5}+\sqrt{1}|}{\sqrt{2}} \\
 &= \frac{\sqrt{5}+1}{\sqrt{2}} = \frac{\sqrt{10}+\sqrt{2}}{2}
 \end{aligned}$$

[6]

$$\begin{aligned}
 x &= \frac{\sqrt{5}+2}{\sqrt{5}-2} & y &= \frac{\sqrt{5}-2}{\sqrt{5}+2} \\
 &= \frac{\sqrt{5}+2}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} & &= \frac{\sqrt{5}-2}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2} \\
 &= 9+4\sqrt{5} & &= 9-4\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 (1) \quad x+y &= (9+4\sqrt{5})+(9-4\sqrt{5}) & (2) \quad xy &= (9+4\sqrt{5})(9-4\sqrt{5}) \\
 &= \mathbf{18} & &= 81-80=\mathbf{1}
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad x^2+y^2 &= (x+y)^2-2xy & (4) \quad \frac{x}{y}+\frac{y}{x} &= \frac{x^2+y^2}{xy} \\
 &= 18^2-2\cdot 1 & &= \frac{322}{1}=\mathbf{322}
 \end{aligned}$$

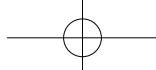
$$\begin{aligned}
 (5) \quad x^3+y^3 &= (x+y)^3-3xy(x+y) & (6) \quad x^4+y^4 &= (x^2+y^2)^2-2x^2y^2 \\
 &= 18^3-3\cdot 1\cdot 18 & &= (x^2+y^2)^2-2(xy)^2 \\
 &= 5832-54=\mathbf{5778} & &= 322^2-2\cdot 1^2 \\
 & & &= 103684-2=\mathbf{103682}
 \end{aligned}$$

[7]

$$\begin{aligned}
 x &= \frac{2-\sqrt{3}}{2+\sqrt{3}} & \frac{1}{x} &= \frac{2+\sqrt{3}}{2-\sqrt{3}} \\
 &= \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} & &= \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} \\
 &= 7-4\sqrt{3} & &= 7+4\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 (1) \quad x+\frac{1}{x} &= (7-4\sqrt{3})+(7+4\sqrt{3}) \\
 &= \mathbf{14}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad x-\frac{1}{x} &= (7-4\sqrt{3})-(7+4\sqrt{3}) & (3) \quad x^2+\frac{1}{x^2} &= \left(x+\frac{1}{x}\right)^2-2\cdot x\cdot \frac{1}{x} \\
 &= -8\sqrt{3} & &= 14^2-2 \\
 & & &= 196-2=\mathbf{194}
 \end{aligned}$$



$$\begin{aligned}
 (4) \quad & x^3 + \frac{1}{x^3} \\
 &= \left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \\
 &= \left(x + \frac{1}{x}\right)^3 - 3 \left(x + \frac{1}{x}\right) \\
 &= 14^3 - 3 \cdot 14 \\
 &= 2744 - 42 = \mathbf{2702}
 \end{aligned}$$

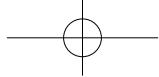
$$\begin{aligned}
 (5) \quad x^3 - \frac{1}{x^3} &= \left(x - \frac{1}{x}\right)^3 + 3 \left(x - \frac{1}{x}\right) \quad (6) \quad \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = x + \frac{1}{x} + 2 \text{ より}, \\
 &= (-8\sqrt{3})^3 + 3 \cdot (-8\sqrt{3}) \\
 &= -1536\sqrt{3} - 24\sqrt{3} \quad \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 \\
 &= -1560\sqrt{3} \quad = x + \frac{1}{x} + 2 \\
 & \qquad \qquad \qquad = 14 + 2 = 16 \\
 &\qquad \qquad \qquad \sqrt{x} + \frac{1}{\sqrt{x}} > 0 \text{ より}, \\
 &\qquad \qquad \qquad \sqrt{x} + \frac{1}{\sqrt{x}} = 4
 \end{aligned}$$

【8】 (1) $\frac{6}{3+\sqrt{3}} = \frac{6}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}} = 3-\sqrt{3}$

$$\begin{aligned}
 1 < \sqrt{3} < 2 \text{ より}, \\
 -2 &< -\sqrt{3} < -1 \\
 3-2 < 3-\sqrt{3} &< -1+3 \\
 1 &< 3-\sqrt{3} < 2
 \end{aligned}$$

なので、 $\frac{6}{3+\sqrt{3}}$ の整数部分 a は、 $a = 1$

$$\begin{aligned}
 (2) \quad \text{整数部分 } a &= 1 \text{ なので}, \\
 b &= \frac{6}{3+\sqrt{3}} - 1 = (3-\sqrt{3}) - 1 = 2-\sqrt{3} \\
 \frac{1}{b} &= \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2+\sqrt{3}
 \end{aligned}$$

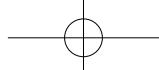


$$\begin{aligned}(3) \quad a^2 + 3ab + b^2 &= (a + b)^2 + ab \\&= (3 - \sqrt{3})^2 + 1 \cdot (2 - \sqrt{3}) \\&= (3 - \sqrt{3})^2 + 2 - \sqrt{3} \\&= 9 - 6\sqrt{3} + 3 + 2 - \sqrt{3} \\&= \mathbf{14 - 7\sqrt{3}}\end{aligned}$$

$$\begin{aligned}(4) \quad (a + b - 1)^3 &= (3 - \sqrt{3} - 1)^3 \\&= (2 - \sqrt{3})^3 \\&= 2^3 - 3 \cdot 2^2 \cdot \sqrt{3} + 3 \cdot 2 \cdot (\sqrt{3})^2 - (\sqrt{3})^3 \\&= 8 - 12\sqrt{3} + 18 - 3\sqrt{3} \\&= \mathbf{26 - 15\sqrt{3}}\end{aligned}$$

$$\begin{aligned}(5) \quad b^2 + \frac{1}{b^2} &= \left(b + \frac{1}{b}\right)^2 - 2 \cdot b \cdot \frac{1}{b} \\&= \left(b + \frac{1}{b}\right)^2 - 2 \\&= (2 - \sqrt{3} + 2 + \sqrt{3})^2 - 2 \\&= 4^2 - 2 \\&= 16 - 2 \\&= \mathbf{14}\end{aligned}$$

$$\begin{aligned}(6) \quad \frac{1}{b^3} - b^3 &= \left(\frac{1}{b} - b\right) \left(\frac{1}{b^2} + b \cdot \frac{1}{b} + b^2\right) \\&= \left(\frac{1}{b} - b\right) \left(\frac{1}{b^2} + 1 + b^2\right) \\&= \left\{(2 + \sqrt{3}) - (2 - \sqrt{3})\right\} (14 + 1) \\&= 2\sqrt{3} \times 15 \\&= \mathbf{30\sqrt{3}}\end{aligned}$$



添削課題

[1] (1) $|x+3| = \begin{cases} x+3 & (x+3 \geq 0, \text{ すなわち, } x \geq -3 \text{ のとき}) \\ -x-3 & (x+3 < 0, \text{ すなわち, } x < -3 \text{ のとき}) \end{cases}$

 $|x-2| = \begin{cases} x-2 & (x-2 \geq 0, \text{ すなわち, } x \geq 2 \text{ のとき}) \\ -x+2 & (x-2 < 0, \text{ すなわち, } x < 2 \text{ のとき}) \end{cases}$

(i) $x \geq 2$ のとき

$$|x+3| - |x-2| = (x+3) - (x-2) \\ = 5$$

(ii) $-3 \leq x < 2$ のとき

$$|x+3| - |x-2| = (x+3) + (x-2) \\ = 2x + 1$$

(iii) $x < -3$ のとき

$$|x+3| - |x-2| = -(x+3) + (x-2) \\ = -5$$

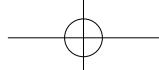
(i)～(iii) より、

$|x+3| - |x-2| = \begin{cases} 5 & (x \geq 2 \text{ のとき}) \\ 2x+1 & (-3 \leq x < 2 \text{ のとき}) \\ -5 & (x < -3 \text{ のとき}) \end{cases}$

(2) ① $(3-2\sqrt{3})^3 = 3^3 - 3 \cdot 3^2 \cdot 2\sqrt{3} + 3 \cdot 3 \cdot (2\sqrt{3})^2 - (2\sqrt{3})^3$
 $= 27 - 54\sqrt{3} + 108 - 24\sqrt{3}$
 $= 135 - 78\sqrt{3}$

② $(\sqrt{2}-\sqrt{3}-\sqrt{5})^2 = 2+3+5-2\sqrt{6}-2\sqrt{10}+2\sqrt{15}$
 $= 10 - 2\sqrt{6} - 2\sqrt{10} + 2\sqrt{15}$

[2] $(9+4\sqrt{5})^n = A, (9-4\sqrt{5})^n = B$ とおくと、
与式 $= (A+B)^2 - (A-B)^2 = 4AB$
 $= 4(9+4\sqrt{5})^n(9-4\sqrt{5})^n$
 $= 4\{(9+4\sqrt{5})(9-4\sqrt{5})\}^n$
 $= 4 \cdot (81-80)^n = 4 \cdot 1^n = 4$



$$[3] \quad \sqrt{1 + \frac{\sqrt{3}}{2}} = \sqrt{\frac{4 + 2\sqrt{3}}{4}} = \frac{\sqrt{3} + 1}{2}$$

ここで、 $1 < \sqrt{3} < 2$ より、

$$1 < \sqrt{3} < 2$$

$$2 < \sqrt{3} + 1 < 3$$

$$\therefore 1 < \frac{\sqrt{3} + 1}{2} < \frac{3}{2}$$

したがって、 $\sqrt{1 + \frac{\sqrt{3}}{2}}$ の整数部分 a は、 $a = 1$.

小数部分 b は、 $b = \frac{\sqrt{3} + 1}{2} - 1 = \frac{\sqrt{3} - 1}{2}$.

よって、 $a + b = \frac{\sqrt{3} + 1}{2}$, $a - b = 1 - \frac{\sqrt{3} - 1}{2} = \frac{3 - \sqrt{3}}{2}$ だから、

$$\begin{aligned} \frac{1}{a+b} + \frac{1}{a-b} &= \frac{2}{\sqrt{3}+1} + \frac{2}{\sqrt{3}(\sqrt{3}-1)} \\ &= \frac{2(3-\sqrt{3}+\sqrt{3}+1)}{\sqrt{3}(\sqrt{3}+1)(\sqrt{3}-1)} \\ &= \frac{2 \cdot 4}{\sqrt{3} \cdot (3-1)} = \frac{4\sqrt{3}}{3} \end{aligned}$$

$$[4] \quad x - 2 = \left(\sqrt{a} + \frac{1}{\sqrt{a}} \right)^2 - 2 = a + \frac{1}{a}$$

また、

$$x^2 - 4x = (x-2)^2 - 4 = \left(a + \frac{1}{a} \right)^2 - 4 = \left(a - \frac{1}{a} \right)^2$$

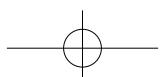
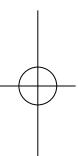
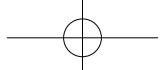
$$\therefore \sqrt{x^2 - 4x} = \left| a - \frac{1}{a} \right|$$

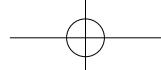
したがって、 $a \geq \frac{1}{a}$ ($a > 0$) のとき、すなわち、 $1 \leq a$ のとき

$$x - 2 + \sqrt{x^2 - 4x} = a + \frac{1}{a} + a - \frac{1}{a} = 2a$$

$a < \frac{1}{a}$ ($a > 0$) のとき、すなわち、 $0 < a < 1$ のとき

$$x - 2 + \sqrt{x^2 - 4x} = a + \frac{1}{a} - \left(a - \frac{1}{a} \right) = \frac{2}{a}$$





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氏名	
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