M1J/M1JS

本科１期６月度

解答

Ｚ会東大進学教室

高１選抜東大数学

高１東大数学

８章　三角関数（３）

問題

【1】(1)

$$\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$
$$= \mathbf{\frac{\sqrt{6}-\sqrt{2}}{4}} \quad \text{(答)}$$

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$
$$= \mathbf{\frac{\sqrt{6}+\sqrt{2}}{4}} \quad \text{(答)}$$

$$\tan 15^\circ = \frac{\sin 15^\circ}{\cos 15^\circ} = \frac{\frac{\sqrt{6}-\sqrt{2}}{4}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{(\sqrt{6}-\sqrt{2})^2}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}$$
$$= \mathbf{2-\sqrt{3}} \quad \text{(答)}$$

＜別解＞

正接の加法定理を用いると

$$\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{4-2\sqrt{3}}{2}$$
$$= \mathbf{2-\sqrt{3}} \quad \text{(答)}$$

(2)

$$\sin 105^\circ = \sin(45^\circ + 60^\circ) = \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ$$
$$= \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}}$$
$$= \mathbf{\frac{\sqrt{2}+\sqrt{6}}{4}} \quad \text{(答)}$$

$$\cos 105^\circ = \cos(45^\circ + 60^\circ) = \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$$
$$= \frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$
$$= \mathbf{\frac{\sqrt{2}-\sqrt{6}}{4}} \quad \text{(答)}$$

$$\tan 105^\circ = \frac{\sin 105^\circ}{\cos 105^\circ} = \frac{\dfrac{\sqrt{2}+\sqrt{6}}{4}}{\dfrac{\sqrt{2}-\sqrt{6}}{4}} = \frac{(\sqrt{2}+\sqrt{6})^2}{(\sqrt{2}-\sqrt{6})(\sqrt{2}+\sqrt{6})}$$
$$= \boldsymbol{-2-\sqrt{3}} \qquad \text{(答)}$$

＜別解＞

正接の加法定理を用いると

$$\tan 105^\circ = \tan(45^\circ + 60^\circ) = \frac{\tan 45^\circ + \tan 60^\circ}{1 - \tan 45^\circ \tan 60^\circ} = \frac{1+\sqrt{3}}{1 - 1 \times \sqrt{3}}$$
$$= \frac{1+\sqrt{3}}{1-\sqrt{3}} = \frac{(1+\sqrt{3})^2}{(1-\sqrt{3})(1+\sqrt{3})} = \frac{4+2\sqrt{3}}{-2}$$
$$= \boldsymbol{-2-\sqrt{3}} \qquad \text{(答)}$$

＜コメント＞

(1) の結果から，次のように導いてもよい．

$$\sin 105^\circ = \sin(90^\circ + 15^\circ) = \cos 15^\circ = \boldsymbol{\frac{\sqrt{6}+\sqrt{2}}{4}} \qquad \text{(答)}$$

$$\cos 105^\circ = \cos(90^\circ + 15^\circ) = -\sin 15^\circ = \boldsymbol{\frac{-\sqrt{6}+\sqrt{2}}{4}} \qquad \text{(答)}$$

$$\tan 105^\circ = \tan(90^\circ + 15^\circ) = -\frac{1}{\tan 15^\circ} = -\frac{1}{2-\sqrt{3}} = \boldsymbol{-2-\sqrt{3}} \qquad \text{(答)}$$

(3)

$$\sin 195^\circ = \sin(135^\circ + 60^\circ) = \sin 135^\circ \cos 60^\circ + \cos 135^\circ \sin 60^\circ$$
$$= \frac{1}{\sqrt{2}} \times \frac{1}{2} + \left(-\frac{1}{\sqrt{2}}\right) \times \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$
$$= \boldsymbol{\frac{\sqrt{2}-\sqrt{6}}{4}} \qquad \text{(答)}$$

$$\cos 195^\circ = \cos(135^\circ + 60^\circ) = \cos 135^\circ \cos 60^\circ - \sin 135^\circ \sin 60^\circ$$
$$= -\frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = \frac{-1-\sqrt{3}}{2\sqrt{2}}$$
$$= \boldsymbol{-\frac{\sqrt{2}+\sqrt{6}}{4}} \qquad \text{(答)}$$

$$\tan 195^\circ = \frac{\sin 195^\circ}{\cos 195^\circ} = \frac{\dfrac{\sqrt{2}-\sqrt{6}}{4}}{-\dfrac{\sqrt{2}+\sqrt{6}}{4}} = \frac{(\sqrt{2}-\sqrt{6})^2}{-(\sqrt{2}+\sqrt{6})(\sqrt{2}-\sqrt{6})}$$
$$= \boldsymbol{2-\sqrt{3}} \qquad \text{(答)}$$

(4)
$$\sin\frac{11}{12}\pi=\sin\left(\frac{2}{3}\pi+\frac{\pi}{4}\right)=\sin\frac{2}{3}\pi\cos\frac{\pi}{4}+\cos\frac{2}{3}\pi\sin\frac{\pi}{4}$$
$$=\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}+\left(-\frac{1}{2}\right)\times\frac{1}{\sqrt{2}}=\frac{\sqrt{3}-1}{2\sqrt{2}}$$
$$=\boldsymbol{\frac{\sqrt{6}-\sqrt{2}}{4}}\qquad(答)$$

$$\cos\frac{11}{12}\pi=\cos\left(\frac{2}{3}\pi+\frac{\pi}{4}\right)=\cos\frac{2}{3}\pi\cos\frac{\pi}{4}-\sin\frac{2}{3}\pi\sin\frac{\pi}{4}$$
$$=\left(-\frac{1}{2}\right)\times\frac{1}{\sqrt{2}}-\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}=\frac{-1-\sqrt{3}}{2\sqrt{2}}$$
$$=\boldsymbol{-\frac{\sqrt{6}+\sqrt{2}}{4}}\qquad(答)$$

$$\tan\frac{11}{12}\pi=\frac{\sin\frac{11}{12}\pi}{\cos\frac{11}{12}\pi}=\frac{\frac{\sqrt{6}-\sqrt{2}}{4}}{-\frac{\sqrt{6}+\sqrt{2}}{4}}=\frac{(\sqrt{6}-\sqrt{2})^2}{-(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}$$
$$=\boldsymbol{\sqrt{3}-2}\qquad(答)$$

(5)
$$\frac{\pi}{8}=\frac{\frac{\pi}{4}}{2}$$

であることから，半角の公式よりそれぞれ

$$\sin^2\frac{\pi}{8}=\sin^2\frac{\frac{\pi}{4}}{2}=\frac{1-\cos\frac{\pi}{4}}{2}=\frac{1-\frac{1}{\sqrt{2}}}{2}=\frac{2-\sqrt{2}}{4}$$
$$\cos^2\frac{\pi}{8}=\cos^2\frac{\frac{\pi}{4}}{2}=\frac{1+\cos\frac{\pi}{4}}{2}=\frac{1+\frac{1}{\sqrt{2}}}{2}=\frac{2+\sqrt{2}}{4}$$

より，$\frac{\pi}{8}$ は第 1 象限の角であることを考慮すると

$$\sin\frac{\pi}{8}=\boldsymbol{\frac{\sqrt{2-\sqrt{2}}}{2}}\qquad(答)$$
$$\cos\frac{\pi}{8}=\boldsymbol{\frac{\sqrt{2+\sqrt{2}}}{2}}\qquad(答)$$
$$\therefore\ \tan\frac{\pi}{8}=\frac{\sin\frac{\pi}{8}}{\cos\frac{\pi}{8}}=\frac{\frac{\sqrt{2-\sqrt{2}}}{2}}{\frac{\sqrt{2+\sqrt{2}}}{2}}=\sqrt{\frac{(2-\sqrt{2})^2}{(2+\sqrt{2})(2-\sqrt{2})}}=\sqrt{3-2\sqrt{2}}$$
$$=\boldsymbol{\sqrt{2}-1}\qquad(答)$$

(6) $67.5^\circ = \dfrac{135^\circ}{2}$

であるから，半角の公式よりそれぞれ

$$\sin^2 67.5^\circ = \sin^2 \frac{135^\circ}{2} = \frac{1-\cos 135^\circ}{2} = \frac{1-\left(-\frac{1}{\sqrt{2}}\right)}{2} = \frac{\sqrt{2}+1}{2\sqrt{2}}$$
$$= \frac{2+\sqrt{2}}{4}$$

$$\cos^2 67.5^\circ = \cos^2 \frac{135^\circ}{2} = \frac{1+\cos 135^\circ}{2} = \frac{1+\left(-\frac{1}{\sqrt{2}}\right)}{2} = \frac{\sqrt{2}-1}{2\sqrt{2}}$$
$$= \frac{2-\sqrt{2}}{4}$$

より，67.5° が第 1 象限の角であることを考慮すると

$$\sin 67.5^\circ = \boldsymbol{\frac{\sqrt{2+\sqrt{2}}}{2}} \qquad \text{(答)}$$
$$\cos 67.5^\circ = \boldsymbol{\frac{\sqrt{2-\sqrt{2}}}{2}} \qquad \text{(答)}$$

$$\therefore\ \tan 67.5^\circ = \frac{\sin 67.5^\circ}{\cos 67.5^\circ} = \frac{\frac{\sqrt{2+\sqrt{2}}}{2}}{\frac{\sqrt{2-\sqrt{2}}}{2}} = \sqrt{\frac{(2+\sqrt{2})^2}{(2-\sqrt{2})(2+\sqrt{2})}}$$
$$= \sqrt{3+2\sqrt{2}} = \boldsymbol{\sqrt{2}+1} \qquad \text{(答)}$$

(7) $\dfrac{41}{12}\pi = 4\pi - \dfrac{7}{12}\pi = 4\pi - \left(\dfrac{\pi}{3}+\dfrac{\pi}{4}\right)$

より

$$\sin \frac{41}{12}\pi = \sin\left\{4\pi - \left(\frac{\pi}{3}+\frac{\pi}{4}\right)\right\}$$
$$= -\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = -\sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= -\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} - \frac{1}{2}\times\frac{1}{\sqrt{2}} = -\frac{\sqrt{3}+1}{2\sqrt{2}}$$
$$= \boldsymbol{-\frac{\sqrt{2}+\sqrt{6}}{4}} \qquad \text{(答)}$$

$$\cos \frac{41}{12}\pi = \cos\left\{4\pi - \left(\frac{\pi}{3}+\frac{\pi}{4}\right)\right\}$$
$$= \cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= \frac{1}{2}\times\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$
$$= \boldsymbol{\frac{\sqrt{2}-\sqrt{6}}{4}} \qquad \text{(答)}$$

$$\tan\frac{41}{12}\pi = \frac{\sin\frac{41}{12}\pi}{\cos\frac{41}{12}\pi} = \frac{-\frac{\sqrt{2}+\sqrt{6}}{4}}{\frac{\sqrt{2}-\sqrt{6}}{4}} = \frac{-(\sqrt{2}+\sqrt{6})^2}{(\sqrt{2}-\sqrt{6})(\sqrt{2}+\sqrt{6})}$$

$$= \mathbf{2+\sqrt{3}} \qquad \text{(答)}$$

(8) $450^\circ = 360^\circ + 90^\circ$

であるから

$\sin 450^\circ = \sin 90^\circ = \mathbf{1}$ (答)
$\cos 450^\circ = \cos 90^\circ = \mathbf{0}$ (答)
$\tan 450^\circ = \tan 90^\circ$ より $\tan 450^\circ$ の値は**存在しない** (答)

【2】(1) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ において，α を $90^\circ-\alpha$ とおきかえると

$\cos\{(90^\circ-\alpha)-\beta\} = \cos(90^\circ-\alpha)\cos\beta + \sin(90^\circ-\alpha)\sin\beta$

ここで，$\sin(90^\circ-\alpha) = \cos\alpha$，$\cos(90^\circ-\alpha) = \sin\alpha$ だから

(左辺) $= \cos\{(90^\circ-(\alpha+\beta)\} = \sin(\alpha+\beta)$
(右辺) $= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

ゆえに

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ 〔証明終〕

(2) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ において，α を $90^\circ-\alpha$ とおきかえると

$\cos\{(90^\circ-\alpha)+\beta\} = \cos(90^\circ-\alpha)\cos\beta - \sin(90^\circ-\alpha)\sin\beta$

よって，(1) と同様にして

(左辺) $= \cos\{(90^\circ-(\alpha-\beta)\} = \sin(\alpha-\beta)$
(右辺) $= \sin\alpha\cos\beta - \cos\alpha\sin\beta$

ゆえに

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ 〔証明終〕

＜コメント＞
(2) は，(1) において，β を $-\beta$ とおきかえて導くこともできる.

【3】

$\sin 5^\circ \sin 125^\circ + \sin 5^\circ \sin 245^\circ + \sin 125^\circ \sin 245^\circ$
$= \sin 5^\circ \sin(120^\circ+5^\circ) + \sin 5^\circ \sin(240^\circ+5^\circ) + \sin(120^\circ+5^\circ)\sin(240^\circ+5^\circ)$

ここで，$a=\sin 5^\circ$，$b=\cos 5^\circ$ とおくと

$$\begin{aligned}
(\text{与式}) =& a\left(\sin 120^\circ\cdot b+\cos 120^\circ\cdot a\right)+a\left(\sin 240^\circ\cdot b+\cos 240^\circ\cdot a\right)\\
&+\left(\sin 120^\circ\cdot b+\cos 120^\circ\cdot a\right)\left(\sin 240^\circ\cdot b+\cos 240^\circ\cdot a\right)\\
=& a\left(\frac{\sqrt{3}}{2}b-\frac{1}{2}a\right)+a\left(-\frac{\sqrt{3}}{2}b-\frac{1}{2}a\right)+\left(\frac{\sqrt{3}}{2}b-\frac{1}{2}a\right)\left(-\frac{\sqrt{3}}{2}b-\frac{1}{2}a\right)\\
=& -\frac{1}{2}a^2-\frac{1}{2}a^2-\frac{1}{4}(\sqrt{3}b-a)(\sqrt{3}b+a)\\
=& -a^2-\frac{1}{4}(3b^2-a^2)\\
=& -\frac{3}{4}(a^2+b^2)\\
=& -\boldsymbol{\frac{3}{4}} \qquad (\text{答})
\end{aligned}$$

＜別解１＞

$$\begin{aligned}
&\sin 5^\circ\sin 125^\circ+\sin 5^\circ\sin 245^\circ+\sin 125^\circ\sin 245^\circ\\
=& -\frac{1}{2}\left\{\cos(5^\circ+125^\circ)-\cos(5^\circ-125^\circ)\right\}-\frac{1}{2}\left\{\cos(5^\circ+245^\circ)-\cos(5^\circ-245^\circ)\right\}\\
&-\frac{1}{2}\left\{\cos(125^\circ+245^\circ)-\cos(125^\circ-245^\circ)\right\}\\
=& -\frac{1}{2}\left\{\cos(120^\circ+10^\circ)-\cos(-120^\circ)\right\}-\frac{1}{2}\left\{\cos 250^\circ-\cos(-240^\circ)\right\}\\
&-\frac{1}{2}\left\{\cos(360^\circ+10^\circ)-\cos(-120^\circ)\right\}\\
=& -\frac{1}{2}\left\{\cos(120^\circ+10^\circ)-\cos 120^\circ\right\}-\frac{1}{2}\left\{\cos(-120^\circ+10^\circ)-\cos 120^\circ\right\}\\
&-\frac{1}{2}\left\{\cos 10^\circ-\cos 120^\circ\right\}\\
=& -\frac{1}{2}\left\{\cos(120^\circ+10^\circ)+\cos(-120^\circ+10^\circ)+\cos 10^\circ-3\cos 120^\circ\right\}\\
=& -\frac{1}{2}\left\{(\cos 120^\circ\cos 10^\circ-\sin 120^\circ\sin 10^\circ)\right.\\
&\left.+(\cos(-120^\circ)\cos 10^\circ-\sin(-120^\circ)\sin 10^\circ)+\cos 10^\circ-3\cos 120^\circ\right\}\\
=& -\frac{1}{2}\left(2\cos 120^\circ\cos 10^\circ+\cos 10^\circ-3\cos 120^\circ\right)\\
=& -\frac{1}{2}\left\{2\cdot\left(-\frac{1}{2}\right)\cdot\cos 10^\circ+\cos 10^\circ-3\cdot\left(-\frac{1}{2}\right)\right\}\\
=& -\boldsymbol{\frac{3}{4}} \qquad (\text{答})
\end{aligned}$$

＜別解２＞

$$\begin{aligned}
&\sin 5^\circ\sin 125^\circ+\sin 5^\circ\sin 245^\circ+\sin 125^\circ\sin 245^\circ\\
=& \sin 125^\circ\times(\sin 5^\circ+\sin 245^\circ)+\sin 5^\circ\sin 245^\circ\\
=& \sin 125^\circ\times\left\{\sin(125^\circ-120^\circ)+\sin(125^\circ+120^\circ)\right\}\\
&+\sin(125^\circ-120^\circ)\sin(125^\circ+120^\circ)\\
=& \sin 125^\circ\times\left\{-\frac{1}{2}(\sqrt{3}\cos 125^\circ+\sin 125^\circ)+\frac{1}{2}(\sqrt{3}\cos 125^\circ-\sin 125^\circ)\right\}\\
&+\left\{-\frac{1}{2}(\sqrt{3}\cos 125^\circ+\sin 125^\circ)\right\}\times\left\{\frac{1}{2}(\sqrt{3}\cos 125^\circ-\sin 125^\circ)\right\}
\end{aligned}$$

ここで，$\cos 125^\circ = a$，$\sin 125^\circ = b$ とすると

$$
\begin{aligned}
(\text{与式}) &= b \times \left\{-\frac{1}{2}(\sqrt{3}a+b) + \frac{1}{2}(\sqrt{3}a-b)\right\} - \frac{1}{2}(\sqrt{3}a+b) \times \frac{1}{2}(\sqrt{3}a-b) \\
&= -\frac{1}{2}b(\sqrt{3}a+b-\sqrt{3}a+b) - \frac{1}{4}(3a^2-b^2) \\
&= -\frac{3}{4}(a^2+b^2) \\
&= -\boldsymbol{\frac{3}{4}} \qquad (\text{答})
\end{aligned}
$$

【4】

$$
\begin{aligned}
&\cos^2\theta - \sin^2\left(\theta+\frac{\pi}{3}\right) + \cos^2\left(\theta+\frac{2}{3}\pi\right) \\
=&\cos^2\theta - \left(\sin\theta\cos\frac{\pi}{3} + \cos\theta\sin\frac{\pi}{3}\right)^2 + \left(\cos\theta\cos\frac{2}{3}\pi - \sin\theta\sin\frac{2}{3}\pi\right)^2 \\
=&\cos^2\theta - \left(\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right)^2 + \left(-\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right)^2 \\
=&\cos^2\theta - \frac{1}{4}\left(\sin^2\theta + 3\cos^2\theta + 2\sqrt{3}\sin\theta\cos\theta\right) \\
&\qquad + \frac{1}{4}\left(\cos^2\theta + 3\sin^2\theta + 2\sqrt{3}\sin\theta\cos\theta\right) \\
=&\cos^2\theta + \frac{1}{2}\left(\sin^2\theta - \cos^2\theta\right) \\
=&\frac{1}{2}(\cos^2\theta + \sin^2\theta) \\
=&\boldsymbol{\frac{1}{2}} \qquad (\text{答})
\end{aligned}
$$

【5】

$$
\left(\tan x - \frac{1}{\tan x}\right)^2 = \left(\tan x + \frac{1}{\tan x}\right)^2 - 4 = \left(\frac{17}{4}\right)^2 - 4 = \frac{225}{16}
$$

$0 < x \leqq \frac{\pi}{4}$ より，$0 < \tan x \leqq 1$ であるから

$$
\tan x - \frac{1}{\tan x} = -\frac{15}{4}
$$

これより

$$
\tan 2x = \frac{2\tan x}{1-\tan^2 x} = 2 \times \frac{1}{\frac{1}{\tan x} - \tan x} = 2 \times \frac{1}{\frac{15}{4}} = \boldsymbol{\frac{8}{15}} \qquad (\text{答})
$$

さらに

$$
\cos^2 2x = \frac{1}{\tan^2 2x + 1} = \frac{1}{\frac{64}{225}+1} = \frac{225}{289}
$$

$0 < x \leqq \frac{\pi}{4}$ より，$0 < 2x \leqq \frac{\pi}{2}$ であるから

$$
\cos 2x = \sqrt{\frac{225}{289}} = \frac{15}{17}
$$

$$
\therefore\ \sin 2x = \tan 2x \times \cos 2x = \frac{8}{15} \times \frac{15}{17} = \frac{8}{17}
$$

であることから

$$(\sin x+\cos x)^2=1+2\sin x\cos x=1+\sin 2x=1+\frac{8}{17}=\frac{25}{17}$$

$0<x\leqq\dfrac{\pi}{4}$ より，$\sin x>0$，$\cos x>0$ であるから

$$\sin x+\cos x=\sqrt{\frac{25}{17}}=\boldsymbol{\frac{5}{17}\sqrt{17}}\qquad\text{(答)}$$

【6】(1)

$$\cos^2\alpha=1-\sin^2\alpha=1-\left(\frac{1}{4}\right)^2=\frac{15}{16}$$

$0<\alpha<\dfrac{\pi}{2}$ より，$\cos\alpha>0$ だから，$\cos\alpha=\dfrac{\sqrt{15}}{4}$

また

$$\sin^2\beta=1-\cos^2\beta=1-\left(-\frac{1}{3}\right)^2=\frac{8}{9}$$

$\dfrac{\pi}{2}<\alpha<\pi$ より，$\sin\beta>0$ だから，$\sin\beta=\dfrac{2\sqrt{2}}{3}$

したがって

$$\begin{aligned}\sin(\alpha+\beta)&=\sin\alpha\cos\beta+\cos\alpha\sin\beta\\&=\frac{1}{4}\times\left(-\frac{1}{3}\right)+\frac{\sqrt{15}}{4}\times\frac{2\sqrt{2}}{3}=-\frac{1}{12}+\frac{\sqrt{30}}{6}\\&=\boldsymbol{\frac{-1+2\sqrt{30}}{12}}\qquad\text{(答)}\end{aligned}$$

$$\begin{aligned}\cos(\alpha+\beta)&=\cos\alpha\cos\beta-\sin\alpha\sin\beta\\&=\frac{\sqrt{15}}{4}\times\left(-\frac{1}{3}\right)-\frac{1}{4}\times\frac{2\sqrt{2}}{3}=-\frac{\sqrt{15}}{12}-\frac{\sqrt{2}}{6}\\&=\boldsymbol{\frac{-\sqrt{15}-2\sqrt{2}}{12}}\qquad\text{(答)}\end{aligned}$$

(2)

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{2+(-2)}{1-2\times(-2)}=\mathbf{0}\qquad\text{(答)}$$

$\tan^2\alpha+1=\dfrac{1}{\cos^2\alpha}$ より，$\cos^2\alpha=\dfrac{1}{2^2+1}=\dfrac{1}{5}$

$0<\alpha<\dfrac{\pi}{2}$ より，$\cos\alpha>0$ だから，$\cos\alpha=\dfrac{1}{\sqrt{5}}$

よって

$$\sin\alpha=\tan\alpha\cos\alpha=2\times\frac{1}{\sqrt{5}}=\frac{2}{\sqrt{5}}$$

$\tan^2\beta + 1 = \dfrac{1}{\cos^2\beta}$ より，$\cos^2\beta = \dfrac{1}{(-2)^2+1} = \dfrac{1}{5}$

$\dfrac{\pi}{2} < \beta < \pi$ より，$\cos\beta < 0$ だから，$\cos\beta = -\dfrac{1}{\sqrt{5}}$

よって

$\sin\beta = \tan\beta\cos\beta = -2 \times \left(-\dfrac{1}{\sqrt{5}}\right) = \dfrac{2}{\sqrt{5}}$

したがって

$$\begin{aligned}\sin(\alpha-\beta) &= \sin\alpha\cos\beta - \cos\alpha\sin\beta \\ &= \frac{2}{\sqrt{5}} \times \left(-\frac{1}{\sqrt{5}}\right) - \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = -\frac{2}{5} - \frac{2}{5} \\ &= -\boldsymbol{\frac{4}{5}} \qquad \text{(答)}\end{aligned}$$

(3)　$\sin\alpha - \sin\beta = \dfrac{1}{2}$ の両辺を 2 乗して

$\sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta = \dfrac{1}{4}$　$\cdots\cdots$①

$\cos\alpha + \cos\beta = \dfrac{1}{3}$ の両辺を 2 乗して

$\cos^2\alpha + 2\cos\alpha\cos\beta + \cos^2\beta = \dfrac{1}{9}$　$\cdots\cdots$②

① + ② より

$(\sin^2\alpha + \cos^2\alpha) + 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta) + (\sin^2\beta + \cos^2\beta) = \dfrac{13}{36}$

$1 + 2\cos(\alpha+\beta) + 1 = \dfrac{13}{36}$

よって，$\cos(\alpha+\beta) = -\boldsymbol{\dfrac{59}{72}}$　(答)

【7】(1)　$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\dfrac{3}{5}\right)^2 = \dfrac{16}{25}$

ここで，$0 < \alpha < \dfrac{\pi}{2}$ より，$\cos\alpha > 0$ だから，$\cos\alpha = \dfrac{4}{5}$ であり，また

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\frac{3}{5}}{\frac{4}{5}} = \dfrac{3}{4}$ である．

したがって

$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2 \times \dfrac{3}{5} \times \dfrac{4}{5} = \boldsymbol{\dfrac{24}{25}}$　(答)

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(\dfrac{4}{5}\right)^2 - \left(\dfrac{3}{5}\right)^2 = \boldsymbol{\dfrac{7}{25}}$　(答)

$\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2} = \boldsymbol{\dfrac{24}{7}}$　(答)

$0 < \alpha < \dfrac{\pi}{2}$ より，$0 < \dfrac{\alpha}{2} < \dfrac{\pi}{4}$ だから，$\sin\dfrac{\alpha}{2} > 0$，$\cos\dfrac{\alpha}{2} > 0$，$\tan\dfrac{\alpha}{2} > 0$ であることに注意して

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} = \frac{1-\dfrac{4}{5}}{2} = \frac{1}{10}$$

よって，$\sin\dfrac{\alpha}{2} = \boldsymbol{\dfrac{1}{\sqrt{10}}}$ (答)

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} = \frac{1+\dfrac{4}{5}}{2} = \frac{9}{10}$$

よって，$\cos\dfrac{\alpha}{2} = \boldsymbol{\dfrac{3}{\sqrt{10}}}$ (答)

$$\tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{1-\dfrac{4}{5}}{1+\dfrac{4}{5}} = \frac{1}{9}$$

よって，$\tan\dfrac{\alpha}{2} = \boldsymbol{\dfrac{1}{3}}$ (答)

(2) $\tan^2\beta + 1 = \dfrac{1}{\cos^2\beta}$ より，$\left(-\dfrac{5}{12}\right)^2 + 1 = \dfrac{1}{\cos^2\beta}$

よって，$\cos^2\beta = \dfrac{144}{169}$

ここで，$\dfrac{\pi}{2} < \beta < \pi$ より，$\cos\beta < 0$ だから，$\cos\beta = -\dfrac{12}{13}$

また，$\sin\beta = \tan\beta\cos\beta = -\dfrac{5}{12} \times \left(-\dfrac{12}{13}\right) = \dfrac{5}{13}$

したがって

$\sin 2\beta = 2\sin\beta\cos\beta = 2 \times \dfrac{5}{13} \times \left(-\dfrac{12}{13}\right) = \boldsymbol{-\dfrac{120}{169}}$ (答)

$\cos 2\beta = \cos^2\beta - \sin^2\beta = \left(-\dfrac{12}{13}\right)^2 - \left(\dfrac{5}{13}\right)^2 = \boldsymbol{\dfrac{119}{169}}$ (答)

$\tan 2\beta = \dfrac{2\tan\beta}{1-\tan^2\beta} = \dfrac{2 \times \left(-\dfrac{5}{12}\right)}{1-\left(-\dfrac{5}{12}\right)^2} = \boldsymbol{-\dfrac{120}{119}}$ (答)

$\dfrac{\pi}{2} < \beta < \pi$ より，$\dfrac{\pi}{4} < \dfrac{\beta}{2} < \dfrac{\pi}{2}$ だから，
$\sin\dfrac{\beta}{2} > 0$，$\cos\dfrac{\beta}{2} > 0$，$\tan\dfrac{\beta}{2} > 0$ であることに注意して

$$\sin^2\frac{\beta}{2} = \frac{1-\cos\beta}{2} = \frac{1-\left(-\dfrac{12}{13}\right)}{2} = \frac{25}{26}$$

よって，$\sin\dfrac{\beta}{2} = \boldsymbol{\dfrac{5}{\sqrt{26}}}$ (答)

$$\cos^2\frac{\beta}{2} = \frac{1+\cos\beta}{2} = \frac{1+\left(-\dfrac{12}{13}\right)}{2} = \frac{1}{26}$$

よって，$\cos\dfrac{\beta}{2}=\dfrac{\mathbf{1}}{\boldsymbol{\sqrt{26}}}$　　(答)

$$\tan^2\frac{\beta}{2}=\frac{1-\cos\beta}{1+\cos\beta}=\frac{1-\left(-\frac{12}{13}\right)}{1+\left(-\frac{12}{13}\right)}=25$$

よって，$\tan\dfrac{\beta}{2}=\mathbf{5}$　　(答)

【8】(1)　2 直線のなす角を θ $\left(0\leqq\theta\leqq\frac{\pi}{2}\right)$ とすると，$3\sqrt{3}\times\dfrac{2}{\sqrt{3}}=6\fallingdotseq-1$ より，$\theta\fallingdotseq\dfrac{\pi}{2}$ だから

$$\tan\theta=\left|\frac{3\sqrt{3}-\frac{2}{\sqrt{3}}}{1+3\sqrt{3}\times\frac{2}{\sqrt{3}}}\right|=\left|\frac{3\sqrt{3}-\frac{2}{3}\sqrt{3}}{1+6}\right|=\frac{\sqrt{3}}{3}=\frac{1}{\sqrt{3}}$$

よって，$\theta=\boldsymbol{\dfrac{\pi}{6}}$　　(答)

(2)　$3x+y+1=0$ より，$y=-3x-1$

$x+2y-1=0$ より，$y=-\dfrac{1}{2}x+\dfrac{1}{2}$

2 直線のなす角を θ $\left(0\leqq\theta\leqq\frac{\pi}{2}\right)$ とすると，$-3\times\left(-\dfrac{1}{2}\right)=\dfrac{3}{2}\fallingdotseq-1$ より，$\theta\fallingdotseq\dfrac{\pi}{2}$ だから

$$\tan\theta=\left|\frac{-3-\left(-\frac{1}{2}\right)}{1+(-3)\times\left(-\frac{1}{2}\right)}\right|=\left|\frac{-\frac{5}{2}}{\frac{5}{2}}\right|=1$$

よって，$\theta=\boldsymbol{\dfrac{\pi}{4}}$　　(答)

【9】(1) $X=\sin x$ とおくと，(ただし，$-1\leqq X\leqq1$)

$$\cos 2x=1-2\sin^2 x=1-2X^2$$

であることから，与えられた方程式は

$$\cos 2x-\sin x=0$$
$$1-2X^2-X=0$$
$$2X^2+X-1=0$$
$$(2X-1)(X+1)=0$$
$$\therefore X=-1,\ \frac{1}{2}$$

よって，$\sin x=-1,\ \dfrac{1}{2}$ より，

$x=\boldsymbol{\dfrac{\pi}{6},\ \dfrac{5}{6}\pi,\ \dfrac{3}{2}\pi}$　**(答)**

(2) $X=\tan x$ とおくと，

$$\tan 2x=\frac{2\tan x}{1-\tan^2 x}=\frac{2X}{1-X^2}$$

となる．よって，

$$\begin{aligned}\tan 2x+\tan x&=0\\ \frac{2X}{1-X^2}+X&=0\\ 2X+X(1-X^2)&=0\\ X^3-3X&=0\\ X\left(X-\sqrt{3}\right)\left(X+\sqrt{3}\right)&=0\end{aligned}$$

よって，$\tan x=0,\ \pm\sqrt{3}$ より，

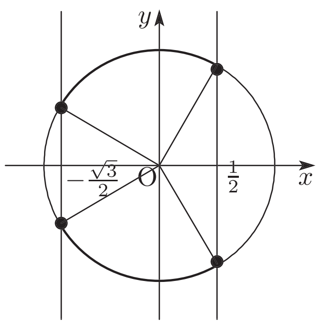
$$x=\mathbf{0},\ \frac{\pi}{3},\ \frac{2}{3}\pi,\ \pi,\ \frac{4}{3}\pi,\ \frac{5}{3}\pi \quad \textbf{(答)}$$

(3) $X=\cos x$ とおくと，(ただし，$-1\leqq X\leqq 1$)

$$\cos 2x=2\cos^2 x-1=2X^2-1$$

であるから，

$$\begin{aligned}2\cos 2x+2\left(\sqrt{3}-1\right)\cos x+2-\sqrt{3}&\leqq 0\\ 2(2X^2-1)+2\left(\sqrt{3}-1\right)X+2-\sqrt{3}&\leqq 0\\ 4X^2+2\left(\sqrt{3}-1\right)X-\sqrt{3}&\leqq 0\\ (2X-1)\left(2X+\sqrt{3}\right)&\leqq 0\\ -\frac{\sqrt{3}}{2}\leqq X&\leqq \frac{1}{2}\end{aligned}$$



よって，$-\frac{\sqrt{3}}{2}\leqq\cos x\leqq\frac{1}{2}$ をみたす x の値の範囲は，右図より，

$$\frac{\pi}{3}\leqq x\leqq\frac{5}{6}\pi,\ \frac{7}{6}\pi\leqq x\leqq\frac{5}{3}\pi \quad \textbf{(答)}$$

(4) $\sin 2x=2\sin x\cos x$ であるので，

$$\begin{aligned}\sin 2x-\sqrt{3}\sin x&>0\\ 2\sin x\cos x-\sqrt{3}\sin x&>0\\ \sin x\left(2\cos x-\sqrt{3}\right)&>0\end{aligned}$$

よって，求める条件は，

「$\sin x>0$　かつ　$\cos x>\frac{\sqrt{3}}{2}$」または

「$\sin x<0$　かつ　$\cos x<\frac{\sqrt{3}}{2}$」

である．

(i) $\sin x > 0$　かつ　$\cos x > \dfrac{\sqrt{3}}{2}$ のとき，

下図 1 より，$0 < x < \dfrac{\pi}{6}$

(ii) $\sin x < 0$　かつ　$\cos x < \dfrac{\sqrt{3}}{2}$ のとき，

下図 2 より，$\pi < x < \dfrac{11}{6}\pi$

(i)，(ii) より，求める x の値の範囲は，

$$\boldsymbol{0 < x < \frac{\pi}{6},\ \pi < x < \frac{11}{6}\pi}$$　**(答)**

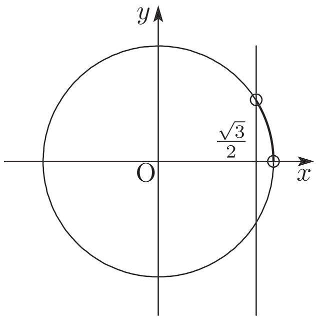


図 1

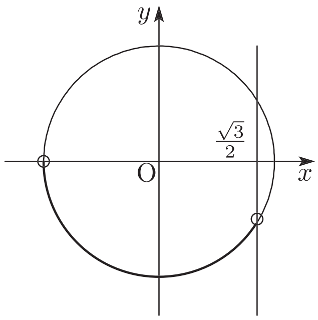


図 2

【10】$72^\circ = 36^\circ \times 2$ より，

$$\cos 72^\circ = \cos(2 \times 36^\circ) = 2\cos^2 36^\circ - 1$$
$$\therefore\ y = 2x^2 - 1 \quad \cdots ①$$

また，$36^\circ = 180^\circ - 72^\circ \times 2$ より

$$\begin{aligned}\cos 36^\circ &= \cos(180^\circ - 72^\circ \times 2) = -\cos(72^\circ \times 2) = -(2\cos^2 72^\circ - 1)\\ &= -2\cos^2 72^\circ + 1\end{aligned}$$
$$\therefore\ x = -2y^2 + 1 \quad \cdots ②$$

①を②に代入して

$$x = -2(2x^2 - 1)^2 + 1$$

整理して

$$\begin{aligned}8x^4 - 8x^2 + x + 1 &= 0\\ 8x^2(x^2 - 1) + (x + 1) &= 0\\ 8x^2(x + 1)(x - 1) + (x + 1) &= 0\\ (x + 1)\{8x^2(x - 1) + 1\} &= 0\\ (x + 1)(8x^3 - 8x^2 + 1) &= 0\\ (x + 1)(2x - 1)(4x^2 - 2x - 1) &= 0\end{aligned}$$
$$\therefore\ x = -1,\ \frac{1}{2},\ \frac{1 \pm \sqrt{5}}{4}$$

$x>0$ より，$x=\dfrac{1}{2},\ \dfrac{1+\sqrt{5}}{4}$

ここで，$x=\dfrac{1}{2}$ のとき，①より

$$y=2\times\left(\frac{1}{2}\right)^2-1=-\frac{1}{2}<0$$

したがって，$y>0$ に不適．よって，$x=\dfrac{1+\sqrt{5}}{4}$

以上から

ア. 2,　イ. 1,　ウ. 1,　エ. 2,　オ. $\boldsymbol{\dfrac{1+\sqrt{5}}{4}}$　　(答)

＜コメント＞

次のように，解を吟味してもよい．

$\cos 45^\circ<\cos 36^\circ<\cos 30^\circ$ より，$\dfrac{1}{\sqrt{2}}<x<\dfrac{\sqrt{3}}{2}$

よって，$x=\dfrac{1+\sqrt{5}}{4}$

【11】$\tan^2\dfrac{x}{2}+1=\dfrac{1}{\cos^2\dfrac{x}{2}}$ より

$$\cos^2\frac{x}{2}=\frac{1}{\tan^2\dfrac{x}{2}+1}=\frac{1}{t^2+1}$$

よって

$$\cos x=\cos\left(2\times\frac{x}{2}\right)=2\cos^2\frac{x}{2}-1=\frac{2}{t^2+1}-1=\frac{1-t^2}{1+t^2}$$

$$\tan x=\tan\left(2\times\frac{x}{2}\right)=\frac{2\tan\dfrac{x}{2}}{1-\tan^2\dfrac{x}{2}}=\frac{2t}{1-t^2}$$

$$\sin x=\tan x\cos x=\frac{2t}{1-t^2}\times\frac{1-t^2}{1+t^2}=\frac{2t}{1+t^2}$$

以上より

$$\boldsymbol{\sin x=\frac{2t}{1+t^2},\quad \cos x=\frac{1-t^2}{1+t^2}}\qquad\text{(答)}$$

添削課題

【1】(1)　(i) $0<\theta<\dfrac{\pi}{2}$ より，$\sin\theta>0$ であるので

$$\sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\left(\frac{2}{3}\right)^2}=\boldsymbol{\frac{\sqrt{5}}{3}}\qquad(答)$$

(ii)

$$\begin{aligned}\sin\left(\theta+\frac{\pi}{3}\right)&=\sin\theta\cos\frac{\pi}{3}+\cos\theta\sin\frac{\pi}{3}\\&=\frac{\sqrt{5}}{3}\cdot\frac{1}{2}+\frac{2}{3}\cdot\frac{\sqrt{3}}{2}\\&=\boldsymbol{\frac{\sqrt{5}+2\sqrt{3}}{6}}\qquad(答)\end{aligned}$$

(2) $\dfrac{5}{12}\pi=\dfrac{\pi}{4}+\dfrac{\pi}{6}$ を考え，

$$\begin{aligned}\cos\frac{5}{12}\pi&=\cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\\&=\cos\frac{\pi}{4}\cos\frac{\pi}{6}-\sin\frac{\pi}{4}\sin\frac{\pi}{6}\\&=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\\&=\boldsymbol{\frac{\sqrt{6}-\sqrt{2}}{4}}\qquad(答)\end{aligned}$$

【2】(1) $0<\alpha<\dfrac{\pi}{2}$，$0<\beta<\dfrac{\pi}{2}$ より，$\cos\alpha>0$，$\cos\beta>0$ であるから，

$$\cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-\left(\frac{2}{3}\right)^2}=\frac{\sqrt{5}}{3}$$

$$\cos\beta=\sqrt{1-\sin^2\beta}=\sqrt{1-\left(\frac{1}{3}\right)^2}=\frac{2\sqrt{2}}{3}$$

以上より，

$$\begin{aligned}\sin(\alpha+\beta)&=\sin\alpha\cos\beta+\cos\alpha\sin\beta\\&=\frac{2}{3}\cdot\frac{2\sqrt{2}}{3}+\frac{\sqrt{5}}{3}\cdot\frac{1}{3}\\&=\boldsymbol{\frac{4\sqrt{2}+\sqrt{5}}{9}}\qquad(答)\\\cos(\alpha+\beta)&=\cos\alpha\cos\beta-\sin\alpha\sin\beta\\&=\frac{\sqrt{5}}{3}\cdot\frac{2\sqrt{2}}{3}-\frac{2}{3}\cdot\frac{1}{3}\\&=\boldsymbol{\frac{2\sqrt{10}-2}{9}}\qquad(答)\end{aligned}$$

(2) $0<\alpha<\dfrac{\pi}{2}$，$0<\beta<\dfrac{\pi}{2}$ より，$\tan\alpha>0$，$\tan\beta>0$ であるから，

$$\tan\alpha=\sqrt{\frac{1}{\cos^2\alpha}-1}=\sqrt{\frac{1}{\left(\frac{2}{3}\right)^2}-1}=\sqrt{\frac{9}{4}-1}=\frac{\sqrt{5}}{2}$$
$$\tan\beta=\sqrt{\frac{1}{\cos^2\beta}-1}=\sqrt{\frac{1}{\left(\frac{\sqrt{3}}{3}\right)^2}-1}=\sqrt{3-1}=\sqrt{2}$$

以上より，

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{\sqrt{5}}{2}+\sqrt{2}}{1-\frac{\sqrt{5}}{2}\cdot\sqrt{2}}=-\boldsymbol{\frac{3\sqrt{2}+2\sqrt{5}}{2}}\qquad\text{(答)}$$

【3】(1) $0<\theta<\dfrac{\pi}{2}$ より，$\cos\theta>0$ であるから，

$$\cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\left(\frac{4}{5}\right)^2}=\sqrt{\frac{9}{25}}=\frac{3}{5}$$

したがって

$$\sin 2\theta=2\sin\theta\cos\theta=2\times\frac{4}{5}\times\frac{3}{5}=\boldsymbol{\frac{24}{25}}\qquad\text{(答)}$$
$$\cos 2\theta=1-2\sin^2\theta=1-2\times\left(\frac{4}{5}\right)^2=-\boldsymbol{\frac{7}{25}}\qquad\text{(答)}$$

また，$0<\theta<\dfrac{\pi}{2}$ より，$0<\dfrac{\theta}{2}<\dfrac{\pi}{4}$ で，$\sin\dfrac{\theta}{2}>0$，$\cos\dfrac{\theta}{2}>0$ であるから，

$$\sin^2\frac{\theta}{2}=\frac{1-\cos\theta}{2}=\frac{1-\frac{3}{5}}{2}=\frac{1}{5}\qquad\therefore\ \sin\frac{\theta}{2}=\boldsymbol{\frac{1}{\sqrt{5}}}\qquad\text{(答)}$$
$$\cos^2\frac{\theta}{2}=\frac{1+\cos\theta}{2}=\frac{1+\frac{3}{5}}{2}=\frac{4}{5}\qquad\therefore\ \cos\frac{\theta}{2}=\boldsymbol{\frac{2}{\sqrt{5}}}\qquad\text{(答)}$$

(2) $t^2=\tan^2\dfrac{\theta}{2}=\dfrac{1-\cos\theta}{1+\cos\theta}$ より，

$$(1+\cos\theta)t^2=1-\cos\theta\iff t^2+t^2\cos\theta=1-\cos\theta$$
$$\iff(t^2+1)\cos\theta=1-t^2$$
$$\therefore\ \boldsymbol{\cos\theta=\frac{1-t^2}{1+t^2}}\qquad\text{(答)}$$

また，倍角の公式より

$$\tan\theta=\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}=\boldsymbol{\frac{2t}{1-t^2}}\qquad\text{(答)}$$

であるから，

$$\sin\theta=\cos\theta\tan\theta=\frac{1-t^2}{1+t^2}\cdot\frac{2t}{1-t^2}=\boldsymbol{\frac{2t}{1+t^2}}\qquad\text{(答)}$$

【4】(1) (i)

$$\sin 4\theta - \sin 2\theta = 2\cos\frac{4\theta+2\theta}{2}\sin\frac{4\theta-2\theta}{2}$$
$$= \boldsymbol{2\cos 3\theta \sin\theta} \quad (答)$$

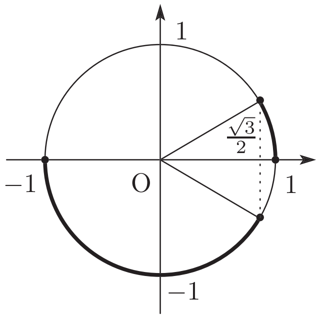
(ii)

$$\sin 3\theta\cos\theta = \frac{1}{2}\{\sin(3\theta+\theta)+\sin(3\theta-\theta)\}$$
$$= \boldsymbol{\frac{1}{2}(\sin 4\theta + \sin 2\theta)} \quad (答)$$

(2) (i) $\sin 2\theta = 2\sin\theta\cos\theta$ より，

$$\sin 2\theta - \sqrt{3}\sin\theta \geqq 0$$
$$\iff 2\sin\theta\cos\theta - \sqrt{3}\sin\theta \geqq 0$$
$$\iff \sin\theta(2\cos\theta - \sqrt{3}) \geqq 0$$

$$\therefore \begin{cases} \sin\theta \geqq 0 \quad かつ \quad \cos\theta \geqq \dfrac{\sqrt{3}}{2} \\ または \\ \sin\theta \leqq 0 \quad かつ \quad \cos\theta \leqq \dfrac{\sqrt{3}}{2} \end{cases}$$



したがって，

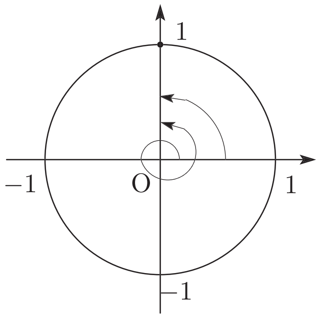
$$\boldsymbol{0 \leqq \theta \leqq \frac{\pi}{6},\ \pi \leqq \theta \leqq \frac{11}{6}\pi} \quad (答)$$

(ii)

$$\sin\left(\theta+\frac{\pi}{6}\right)\cos\left(\theta-\frac{\pi}{3}\right)$$
$$=\frac{1}{2}\left[\sin\left\{\left(\theta+\frac{\pi}{6}\right)+\left(\theta-\frac{\pi}{3}\right)\right\}+\sin\left\{\left(\theta+\frac{\pi}{6}\right)-\left(\theta-\frac{\pi}{3}\right)\right\}\right]$$
$$=\frac{1}{2}\left\{\sin\left(2\theta-\frac{\pi}{6}\right)+\sin\frac{\pi}{2}\right\}$$

より，与えられた方程式は，

$$\sin\left(\theta+\frac{\pi}{6}\right)\cos\left(\theta-\frac{\pi}{3}\right) = 1$$
$$\frac{1}{2}\left\{\sin\left(2\theta-\frac{\pi}{6}\right)+\sin\frac{\pi}{2}\right\} = 1$$
$$\frac{1}{2}\left\{\sin\left(2\theta-\frac{\pi}{6}\right)+1\right\} = 1$$
$$\therefore \sin\left(2\theta-\frac{\pi}{6}\right) = 1$$



$-\dfrac{\pi}{6} \leqq 2\theta - \dfrac{\pi}{6} < \dfrac{23}{6}\pi$ より，

$$2\theta - \frac{\pi}{6} = \frac{\pi}{2},\ \frac{5}{2}\pi$$
$$\therefore \boldsymbol{\theta = \frac{\pi}{3},\ \frac{4}{3}\pi} \quad (答)$$

９章　三角関数（４）

問題

【1】(1) P$(\sqrt{3}, 1)$ とすると

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = 2,\ \alpha = \frac{\pi}{6}$$

だから

$$\sqrt{3}\sin\theta + \cos\theta = \boldsymbol{2\sin\left(\theta + \frac{\pi}{6}\right)} \qquad \text{(答)}$$

(2) P$(-1, 1)$ とすると

$$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2},\ \alpha = \frac{3}{4}\pi$$

だから

$$-\sin\theta + \cos\theta = \boldsymbol{\sqrt{2}\sin\left(\theta + \frac{3}{4}\pi\right)} \qquad \text{(答)}$$

(3) P$(2, -3)$ とすると，$r = \sqrt{2^2 + (-3)^2} = \sqrt{13}$ だから

$$2\sin\theta - 3\cos\theta = \boldsymbol{\sqrt{13}\sin(\theta + \alpha)}$$

$$\boldsymbol{\left(\text{ただし，} \sin\alpha = -\frac{3}{\sqrt{13}},\ \cos\alpha = \frac{2}{\sqrt{13}}\right)} \qquad \text{(答)}$$

(4)
$$\begin{aligned}\sin\left(\theta + \frac{\pi}{6}\right) - \cos\theta &= \sin\theta\cos\frac{\pi}{6} + \cos\theta\sin\frac{\pi}{6} - \cos\theta \\ &= \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta - \cos\theta \\ &= \frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\end{aligned}$$

P$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ とすると

$$r = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = 1,\ \alpha = \frac{11}{6}\pi$$

だから

$$\begin{aligned}\sin\left(\theta + \frac{\pi}{6}\right) - \cos\theta &= \frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta \\ &= \boldsymbol{\sin\left(\theta + \frac{11}{6}\pi\right)} \qquad \text{(答)}\end{aligned}$$

(5)
$$\begin{aligned}\sqrt{2}\sin\theta + 2\sin\left(\theta + \frac{\pi}{4}\right) &= \sqrt{2}\sin\theta + 2\left(\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right) \\ &= \sqrt{2}\sin\theta + \frac{2}{\sqrt{2}}\sin\theta + \frac{2}{\sqrt{2}}\cos\theta \\ &= 2\sqrt{2}\sin\theta + \sqrt{2}\cos\theta\end{aligned}$$

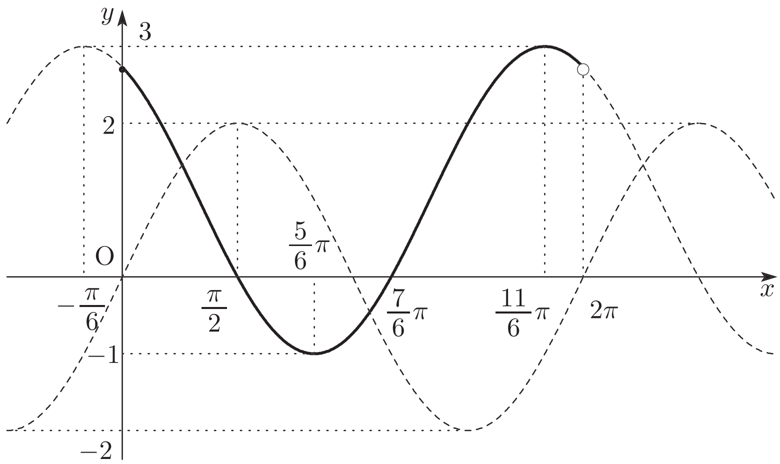
$P(2\sqrt{2}, \sqrt{2})$ とすると，$r = \sqrt{(2\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{10}$ だから

$$\begin{aligned}\sqrt{2}\sin\theta + 2\sin\left(\theta + \frac{\pi}{4}\right) &= 2\sqrt{2}\sin\theta + \sqrt{2}\cos\theta \\ &= \boldsymbol{\sqrt{10}\sin(\theta + \alpha)} \\ &\left(\text{ただし，}\boldsymbol{\sin\alpha = \frac{1}{\sqrt{5}},\ \cos\alpha = \frac{2}{\sqrt{5}}}\right)\end{aligned}$$ (答)

【2】(1) $f(x) = -\sin x + \sqrt{3}\cos x + 1$ とおく．

$$y = f(x) = 2\sin\left(x + \frac{2}{3}\pi\right) + 1$$

よって，$y = f(x)$ のグラフは，$y = 2\sin x$ のグラフを x 軸の正方向に $-\frac{2}{3}\pi$，y 軸の正方向に 1 平行移動したグラフだから，次の実線部分である．



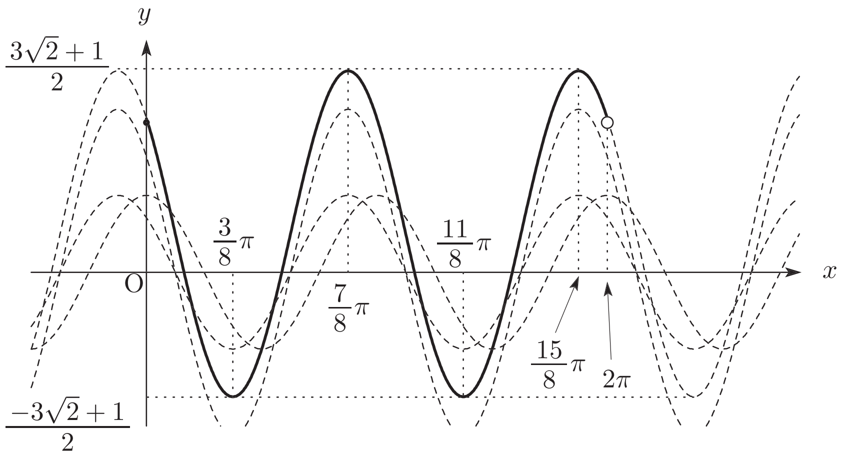
(答)

(2) $f(x) = \cos^2 x + \cos 2x - 3\sin x\cos x$ とおく．

$$\begin{aligned}y &= f(x) \\ &= \frac{1 + \cos 2x}{2} + \cos 2x - \frac{3}{2}\sin 2x \\ &= \frac{3}{2}(\cos 2x - \sin 2x) + \frac{1}{2} = \frac{3}{2}\sqrt{2}\cos\left(2x + \frac{\pi}{4}\right) + \frac{1}{2}\end{aligned}$$

であるから，まず，$y = \cos 2x$ のグラフを x 軸の正方向に $-\frac{\pi}{8}$ 平行移動して $y = \cos\left(2x + \frac{\pi}{4}\right)$ とし，次に y 軸方向に $\frac{3}{2}\sqrt{2}$ 倍して $y = \frac{3}{2}\sqrt{2}\cos\left(2x + \frac{\pi}{4}\right)$ とし，さらに y 軸の正方向に $\frac{1}{2}$ 平行移動したグラフが $y = \frac{3}{2}\sqrt{2}\cos\left(2x + \frac{\pi}{4}\right) + \frac{1}{2}$ である．

したがって，グラフは次の実線部分である．

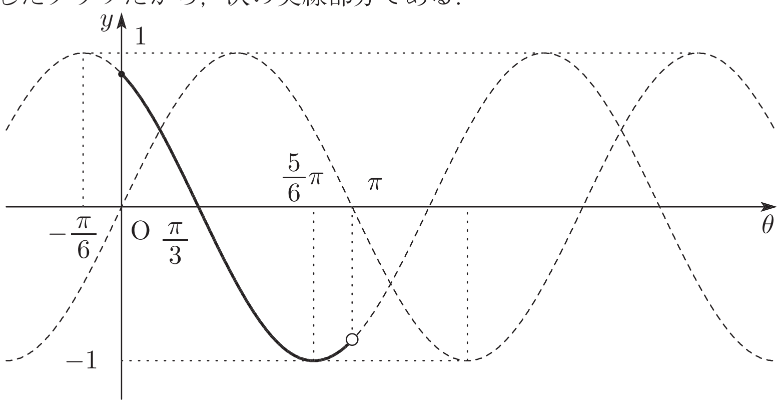


(答)

(3) $f(\theta)=\sqrt{3}\sin\left(\theta+\frac{\pi}{6}\right)-2\sin\theta$ とおく．

$$\begin{aligned} y &= f(\theta) \\ &= \sqrt{3}\sin\left(\theta+\frac{\pi}{6}\right)-2\sin\theta \\ &= \sqrt{3}\left(\sin\theta\cos\frac{\pi}{6}+\cos\theta\sin\frac{\pi}{6}\right)-2\sin\theta \\ &= \sqrt{3}\left(\frac{\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta\right)-2\sin\theta \\ &= \frac{3}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta-2\sin\theta \\ &= -\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta \\ &= \sin\left(\theta+\frac{2}{3}\pi\right) \end{aligned}$$

よって，$y=f(\theta)$ のグラフは，$y=\sin\theta$ のグラフを θ 軸の正方向に $-\frac{2}{3}\pi$ 平行移動したグラフだから，次の実線部分である．



(答)

【3】(1) $\sin\theta+\sqrt{3}\cos\theta=2\sin\left(\theta+\frac{\pi}{3}\right)$

よって

$2\sin\left(\theta+\frac{\pi}{3}\right)=1$

$\sin\left(\theta+\frac{\pi}{3}\right)=\frac{1}{2}$

ここで，$\frac{\pi}{3}\leqq\theta+\frac{\pi}{3}<\frac{7}{3}\pi$ より，$\theta+\frac{\pi}{3}=\frac{5}{6}\pi,\ \frac{13}{6}\pi$

したがって，$\theta=\boldsymbol{\frac{\pi}{2},\ \frac{11}{6}\pi}$　(答)

(2)
$$\sin\left(\theta-\frac{5}{6}\pi\right)+\cos\theta$$
$$=\sin\theta\cos\frac{5}{6}\pi-\cos\theta\sin\frac{5}{6}\pi+\cos\theta=-\frac{\sqrt{3}}{2}\sin\theta-\frac{1}{2}\cos\theta+\cos\theta$$
$$=-\frac{\sqrt{3}}{2}\sin\theta+\frac{1}{2}\cos\theta=\sin\left(\theta+\frac{5}{6}\pi\right)$$

よって，$\sin\left(\theta+\frac{5}{6}\pi\right)=0$

ここで，$\frac{5}{6}\pi\leqq\theta+\frac{5}{6}\pi<\frac{17}{6}\pi$ より，$\theta+\frac{5}{6}\pi=\pi,\ 2\pi$

したがって，$\theta=\boldsymbol{\frac{\pi}{6},\ \frac{7}{6}\pi}$　(答)

(3)
$$\sin^3\theta+\cos^3\theta=(\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)$$
$$=(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)$$

よって，$(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)=0$

ここで，$\sin\theta+\cos\theta=\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)$

また，$1-\sin\theta\cos\theta=1-\frac{1}{2}\sin 2\theta$ だから，$-1\leqq\sin 2\theta\leqq 1$ より，

$\frac{1}{2}\leqq 1-\sin\theta\cos\theta\leqq\frac{3}{2}$ だから，$1-\sin\theta\cos\theta\neq 0$

ゆえに，$\sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right)=0$ より，$\sin\left(\theta+\frac{\pi}{4}\right)=0$

$\frac{\pi}{4}\leqq\theta+\frac{\pi}{4}<\frac{9}{4}\pi$ より，$\theta+\frac{\pi}{4}=\pi,\ 2\pi$

したがって，$\theta=\boldsymbol{\frac{3}{4}\pi,\ \frac{7}{4}\pi}$　(答)

(4) 積→和の公式より

$$\sin\left(\theta+\frac{\pi}{3}\right)\cos\left(\theta-\frac{\pi}{6}\right)=1$$
$$\frac{1}{2}\left\{\sin\left\{\left(\theta+\frac{\pi}{3}\right)+\left(\theta-\frac{\pi}{6}\right)\right\}+\sin\left\{\left(\theta+\frac{\pi}{3}\right)-\left(\theta-\frac{\pi}{6}\right)\right\}\right\}=1$$
$$\frac{1}{2}\left\{\sin\left(2\theta+\frac{\pi}{6}\right)+1\right\}=1$$
$$\sin\left(2\theta+\frac{\pi}{6}\right)=1$$

ここで，$\dfrac{\pi}{6} \leqq 2\theta + \dfrac{\pi}{6} < \dfrac{25}{6}\pi$ より，$2\theta + \dfrac{\pi}{6} = \dfrac{\pi}{2}, \dfrac{5}{2}\pi$

よって，$\theta = \boldsymbol{\dfrac{\pi}{6}, \dfrac{7}{6}\pi}$　　(答)

＜別解＞

角を $\theta - \dfrac{\pi}{6}$ に統一すると

$\sin\left(\theta + \dfrac{\pi}{3}\right)\cos\left(\theta - \dfrac{\pi}{6}\right) = 1$

$\sin\left\{\left(\theta - \dfrac{\pi}{6}\right) + \dfrac{\pi}{2}\right\}\cos\left(\theta - \dfrac{\pi}{6}\right) = 1$

$\cos^2\left(\theta - \dfrac{\pi}{6}\right) = 1$

$\cos\left(\theta - \dfrac{\pi}{6}\right) = \pm 1$

ここで，$-\dfrac{\pi}{6} \leqq \theta - \dfrac{\pi}{6} < \dfrac{11}{6}\pi$ より，$\theta - \dfrac{\pi}{6} = 0, \pi$

よって，$\theta = \boldsymbol{\dfrac{\pi}{6}, \dfrac{7}{6}\pi}$　　(答)

【4】(1)

$$\begin{aligned}\sin x + \sin 2x + \sin 3x &= 0\\ (\sin x + \sin 3x) + \sin 2x &= 0\\ 2\sin 2x\cos(-x) + \sin 2x &= 0\\ 2\sin 2x\cos x + \sin 2x &= 0\\ \sin 2x(2\cos x + 1) &= 0\end{aligned}$$

$\sin 2x = 0$ のとき，$0 \leqq 2x \leqq 2\pi$ より，$2x = 0$，π，2π だから，
$x = 0$，$\dfrac{\pi}{2}$，π

$2\cos x + 1 = 0$ のとき，$\cos x = -\dfrac{1}{2}$

よって，$x = \dfrac{2}{3}\pi$

以上より，$x = \boldsymbol{0,\ \dfrac{\pi}{2},\ \dfrac{2}{3}\pi,\ \pi}$　　(答)

＜別解＞

2 倍角，3 倍角の公式を用いて

$$\begin{aligned}\sin x + 2\sin x\cos x + (3\sin x - 4\sin^3 x) &= 0\\ 4\sin^3 x - 2\sin x\cos x - 4\sin x &= 0\\ 2\sin x(2\sin^2 x - \cos x - 2) &= 0\\ 2\sin x\{2(1 - \cos^2 x) - \cos x - 2\} &= 0\\ 2\sin x(-2\cos^2 x - \cos x) &= 0\\ -2\sin x\cos x(2\cos x + 1) &= 0\end{aligned}$$

よって

$\sin x = 0$ より，$x = 0$，π

$\cos x = 0$ より，$x = \dfrac{\pi}{2}$

$\cos x = -\dfrac{1}{2}$ より，$x = \dfrac{2}{3}\pi$

以上より，$x = \mathbf{0},\ \boldsymbol{\dfrac{\pi}{2}},\ \boldsymbol{\dfrac{2}{3}\pi},\ \boldsymbol{\pi}$　　(答)

(2)　半角の公式，2 倍角の公式より

$$\cos^2\theta + \sqrt{3}\sin\theta\cos\theta = 1$$
$$\frac{1+\cos 2\theta}{2} + \sqrt{3}\times\frac{\sin 2\theta}{2} = 1$$
$$\sqrt{3}\sin 2\theta + \cos 2\theta = 1$$
$$2\sin\left(2\theta + \frac{\pi}{6}\right) = 1$$
$$\sin\left(2\theta + \frac{\pi}{6}\right) = \frac{1}{2}$$

$0 \leqq \theta < 2\pi$ より，$\dfrac{\pi}{6} \leqq 2\theta + \dfrac{\pi}{6} < \dfrac{25}{6}\pi$

よって，$2\theta + \dfrac{\pi}{6} = \dfrac{\pi}{6},\ \dfrac{5}{6}\pi,\ \dfrac{13}{6}\pi,\ \dfrac{17}{6}\pi$ より

$\boldsymbol{\theta = 0,\ \dfrac{\pi}{3},\ \pi,\ \dfrac{4}{3}\pi}$　　(答)

＜別解＞

明らかに，$\cos\theta \fallingdotseq 0$ より，両辺を $\cos^2\theta$ で割ると

$$1 + \sqrt{3}\times\frac{\sin\theta}{\cos\theta} = \frac{1}{\cos^2\theta}$$
$$1 + \sqrt{3}\tan\theta = \tan^2\theta + 1$$
$$\tan^2\theta - \sqrt{3}\tan\theta = 0$$
$$\tan\theta(\tan\theta - \sqrt{3}) = 0$$

$\tan\theta = 0$ のとき，$\theta = 0,\ \pi$

$\tan\theta = \sqrt{3}$ のとき，$\theta = \dfrac{\pi}{3},\ \dfrac{4}{3}\pi$

以上より，$\theta = \mathbf{0},\ \boldsymbol{\dfrac{\pi}{3}},\ \boldsymbol{\pi},\ \boldsymbol{\dfrac{4}{3}\pi}$　　(答)

【5】(1)

$$\sqrt{3}\sin\theta + \cos\theta > \sqrt{2}$$
$$2\sin\left(\theta + \frac{\pi}{6}\right) > \sqrt{2}$$
$$\sin\left(\theta + \frac{\pi}{6}\right) > \frac{\sqrt{2}}{2}$$

$0 \leqq \theta < 2\pi$ より，$\dfrac{\pi}{6} \leqq \theta + \dfrac{\pi}{6} < \dfrac{13}{6}\pi$

よって，$\dfrac{\pi}{4} < \theta + \dfrac{\pi}{6} < \dfrac{3}{4}\pi$

したがって，$\boldsymbol{\dfrac{\pi}{12} < \theta < \dfrac{7}{12}\pi}$　　(答)

(2) $\sin\left(\frac{3}{2}\pi-\theta\right)=\sin\left\{(\pi-\theta)+\frac{\pi}{2}\right\}$

$$=\cos(\pi-\theta)$$
$$=-\cos\theta$$

よって

$$\sin\theta+\sqrt{3}\sin\left(\frac{3}{2}\pi-\theta\right)\geqq 1$$
$$\sin\theta-\sqrt{3}\cos\theta\geqq 1$$
$$2\sin\left(\theta+\frac{5}{3}\pi\right)\geqq 1$$
$$\sin\left(\theta+\frac{5}{3}\pi\right)\geqq \frac{1}{2}$$

$\frac{5}{3}\pi\leqq\theta+\frac{5}{3}\pi<\frac{11}{3}\pi$ より

$\frac{13}{6}\pi\leqq\theta+\frac{5}{3}\pi\leqq\frac{17}{6}\pi$

したがって，$\boldsymbol{\frac{\pi}{2}\leqq\theta\leqq\frac{7}{6}\pi}$ (答)

(3) 3倍角の公式より

$2\cos x+\cos 3x>0$

$2\cos x+4\cos^3 x-3\cos x>0$

$4\cos^3 x-\cos x>0$

$\cos x(2\cos x+1)(2\cos x-1)>0$

$\cos x=0$ では与式は不成立であるから，$\cos x\fallingdotseq 0$ である.

(i) $\cos x>0$ のとき

つまり，$0\leqq x<\frac{\pi}{2}$ のとき，$2\cos x+1>0$ より

$$2\cos x-1>0$$
$$\cos x>\frac{1}{2}$$
$$\therefore 0\leqq x<\frac{\pi}{3}$$

(ii) $\cos x<0$ のとき

つまり，$\frac{\pi}{2}<x<\pi$ のとき，$2\cos x-1<0$ より

$$2\cos x+1>0$$
$$\cos x>-\frac{1}{2}$$
$$\therefore \frac{\pi}{2}<x<\frac{2}{3}\pi$$

よって，$\boldsymbol{0\leqq x<\frac{\pi}{3},\ \frac{\pi}{2}<x<\frac{2}{3}\pi}$ (答)

(4) $\cos^2\theta - 3\cos\theta - \sin^2\theta + 3\sin\theta \geqq 0$

$(\cos^2\theta - \sin^2\theta) - 3(\cos\theta - \sin\theta) \geqq 0$

$(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) - 3(\cos\theta - \sin\theta) \geqq 0$

$(\cos\theta - \sin\theta)(\cos\theta + \sin\theta - 3) \geqq 0$

ここで

$\cos\theta + \sin\theta - 3 = \sqrt{2}\sin\left(\theta + \dfrac{\pi}{4}\right) - 3 \leqq \sqrt{2} - 3 < 0$

よって，$\cos\theta - \sin\theta \leqq 0$ より

$\sqrt{2}\sin\left(\theta + \dfrac{3}{4}\pi\right) \leqq 0$

ゆえに，$\sin\left(\theta + \dfrac{3}{4}\pi\right) \leqq 0$ であり，$\dfrac{3}{4}\pi \leqq \theta + \dfrac{3}{4}\pi < \dfrac{11}{4}\pi$ であることより，

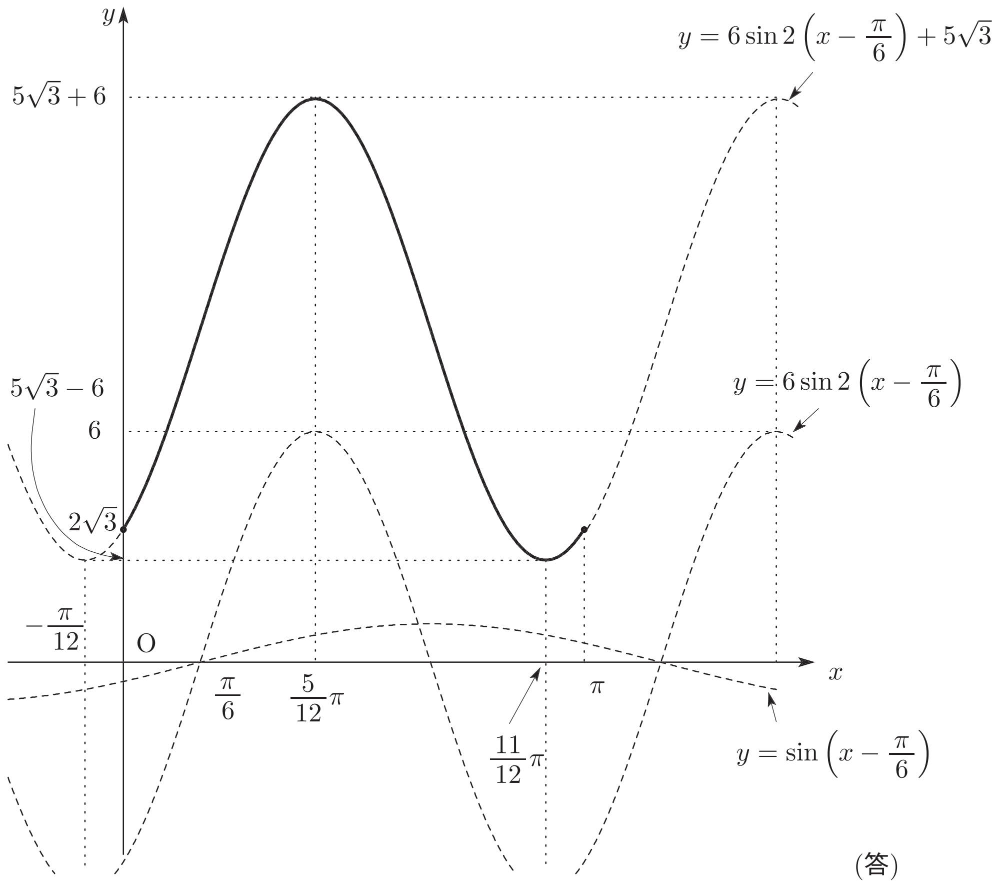
$\pi \leqq \theta + \dfrac{3}{4}\pi \leqq 2\pi$

$\therefore \boldsymbol{\dfrac{\pi}{4} \leqq \theta \leqq \dfrac{5}{4}\pi}$ (答)

【6】半角の公式，2 倍角の公式より

$$\begin{aligned}
y &= 8\sqrt{3}\sin^2 x + 6\sin x\cos x + 2\sqrt{3}\cos^2 x \\
&= 6\sqrt{3} \times \frac{1-\cos 2x}{2} + 3 \times \sin 2x + 2\sqrt{3}(\sin^2 x + \cos^2 x) \\
&= 3\sqrt{3}(1-\cos 2x) + 3\sin 2x + 2\sqrt{3} \\
&= 3\sin 2x - 3\sqrt{3}\cos 2x + 5\sqrt{3} \\
&= 6\sin\left(2x - \frac{\pi}{3}\right) + 5\sqrt{3} \\
&= 6\sin 2\left(x - \frac{\pi}{6}\right) + 5\sqrt{3}
\end{aligned}$$

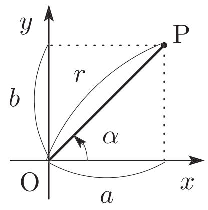
である．この関数のグラフは，$y = \sin x$ のグラフを x 軸の正方向に $\frac{\pi}{6}$ 平行移動して $y = \sin\left(x - \frac{\pi}{6}\right)$ とし，さらに，x 軸方向に $\frac{1}{2}$ 倍，y 軸方向に 6 倍して $y = 6\sin 2\left(x - \frac{\pi}{6}\right)$ とし，最後に，y 軸の正方向に $5\sqrt{3}$ 平行移動したものである．
よって，グラフは次のとおり．



(答)

添削課題

【1】点 $\mathrm{P}(a, b)$ に対して，$\mathrm{OP}=r$，動径 OP が x 軸正の向きとなす角を α とすると



$$a=\boxed{\boldsymbol{r\cos\alpha}},\qquad b=\boxed{\boldsymbol{r\sin\alpha}}\qquad\cdots\cdots①$$

とおけるので，加法定理より

$$\begin{aligned}a\sin\theta+b\cos\theta&=\boxed{\boldsymbol{r\cos\alpha}}\sin\theta+\boxed{\boldsymbol{r\sin\alpha}}\cos\theta\\&=r\sin\theta\cos\alpha+r\cos\theta\sin\alpha\\&=r\left(\boxed{\boldsymbol{\sin\theta\cos\alpha}}+\boxed{\boldsymbol{\cos\theta\sin\alpha}}\right)\\&=r\boxed{\boldsymbol{\sin(\theta+\alpha)}}\end{aligned}$$

ここで

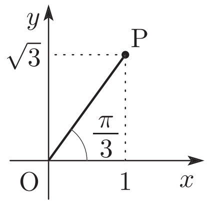
$$r=\boxed{\boldsymbol{\sqrt{a^2+b^2}}}$$

であるから，① より

$$\sin\alpha=\frac{b}{r}=\boxed{\boldsymbol{\frac{b}{\sqrt{a^2+b^2}}}},$$

$$\cos\alpha=\frac{a}{r}=\boxed{\boldsymbol{\frac{a}{\sqrt{a^2+b^2}}}}$$

【2】(1) $\mathrm{P}(1,\sqrt{3})$ とすると



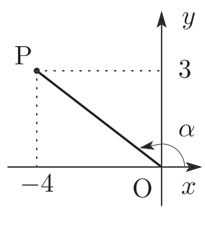
$$\mathrm{OP}=\sqrt{1^2+(\sqrt{3})^2}=2$$

OP と x 軸の正の向きとのなす角は，$\dfrac{\pi}{3}$

よって

$$\sin\theta+\sqrt{3}\cos\theta=\boldsymbol{2\sin\left(\theta+\frac{\pi}{3}\right)}\qquad \text{(答)}$$

(2) $\mathrm{P}(-4, 3)$ とすると



$$\mathrm{OP}=\sqrt{(-4)^2+3^2}=5$$

OP と x 軸の正の向きとのなす角を α とおくと

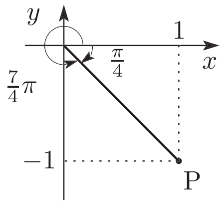
$$-4\sin\theta+3\cos\theta=\boldsymbol{5\sin(\theta+\alpha)}$$

$$\left(\text{ただし，}\boldsymbol{\sin\alpha=\frac{3}{5},\ \cos\alpha=-\frac{4}{5}}\right)\qquad\text{(答)}$$

【3】P(1, −1) とすると

$$\mathrm{OP}=\sqrt{1^2+(-1)^2}=\sqrt{2}$$

OP と x 軸の正の向きとのなす角は，$\dfrac{7}{4}\pi$

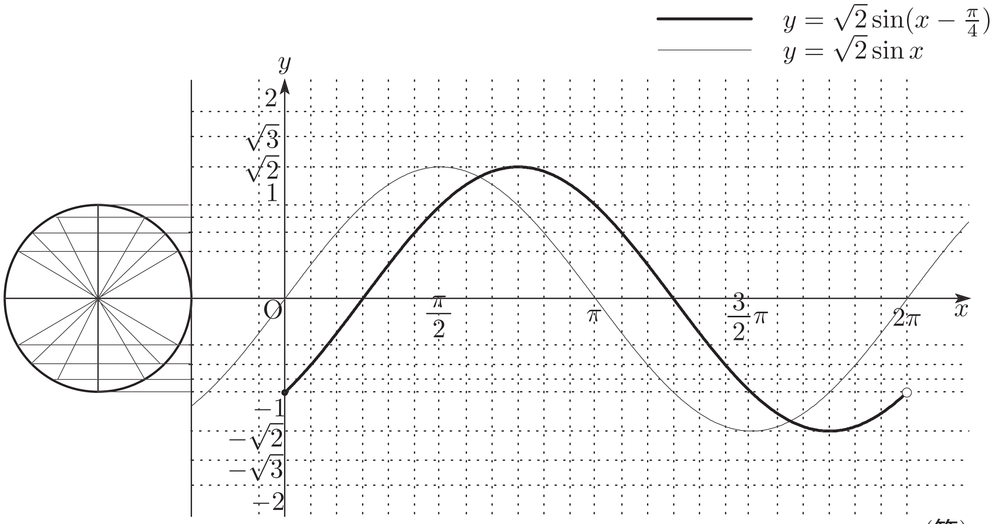


よって

$$y=\sin x-\cos x=\sqrt{2}\sin\left(x+\frac{7}{4}\pi\right)=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$$

したがって，$y=\sqrt{2}\sin\left(x-\dfrac{\pi}{4}\right)$ のグラフは，$y=\sqrt{2}\sin x$

のグラフを x 軸正の方向に $\dfrac{\pi}{4}$ だけ平行移動したグラフだから，次のようになる．



(答)

【4】(1) 2 倍角の公式より

$$4\sin\theta\cos\theta - 2\sin\theta - 2\cos\theta + 1 = 0$$

左辺を因数分解して

$$(2\sin\theta - 1)(2\cos\theta - 1) = 0$$
$$\therefore\ \sin\theta = \frac{1}{2} \quad \text{または} \quad \cos\theta = \frac{1}{2}$$

$0 \leqq \theta < 2\pi$ より

$$\boldsymbol{\theta = \frac{\pi}{6},\ \frac{\pi}{3},\ \frac{5}{6}\pi,\ \frac{5}{3}\pi} \qquad \text{(答)}$$

(2) 和積の公式より

$$2\cos\left(\frac{3}{2}\theta + \frac{\pi}{6}\right)\sin\left(-\frac{\theta}{2} - \frac{\pi}{6}\right) = 0$$
$$\therefore\ \cos\left(\frac{3}{2}\theta + \frac{\pi}{6}\right) = 0 \quad \text{または} \quad \sin\left(-\frac{\theta}{2} - \frac{\pi}{6}\right) = 0$$

$0 \leqq \theta < 2\pi$ より

$$\frac{\pi}{6} \leqq \frac{3}{2}\theta + \frac{\pi}{6} < \frac{19}{6}\pi,\ -\frac{7}{6}\pi < -\frac{\theta}{2} - \frac{\pi}{6} \leqq -\frac{\pi}{6}$$

であるから

$$\frac{3}{2}\theta + \frac{\pi}{6} = \frac{\pi}{2},\ \frac{3}{2}\pi,\ \frac{5}{2}\pi \quad \text{または} \quad -\frac{\theta}{2} - \frac{\pi}{6} = -\pi$$
$$\therefore\ \boldsymbol{\theta = \frac{2}{9}\pi,\ \frac{8}{9}\pi,\ \frac{14}{9}\pi,\ \frac{5}{3}\pi} \qquad \text{(答)}$$

(3) 半角，2 倍角の公式より

$$4 \cdot \frac{1 + \cos 2\theta}{2} - \sin 2\theta + 2 \cdot \frac{1 - \cos 2\theta}{2} \leqq 2$$
$$3 + \cos 2\theta - \sin 2\theta \leqq 2$$
$$\therefore\ \sin 2\theta - \cos 2\theta \geqq 1$$

さらに，三角関数の合成を用いると

$$\sqrt{2}\sin\left(2\theta - \frac{\pi}{4}\right) \geqq 1$$
$$\therefore\ \sin\left(2\theta - \frac{\pi}{4}\right) \geqq \frac{1}{\sqrt{2}}$$

ここで，$0 \leqq \theta < \pi$ より，$-\frac{\pi}{4} \leqq 2\theta - \frac{\pi}{4} < \frac{7}{4}\pi$ であるから

$$\frac{\pi}{4} \leqq 2\theta - \frac{\pi}{4} \leqq \frac{3}{4}\pi$$
$$\therefore\ \boldsymbol{\frac{\pi}{4} \leqq \theta \leqq \frac{\pi}{2}} \qquad \text{(答)}$$

問題

【1】(1) $64=2^6$ より，$\pm\mathbf{2}$　**(答)**

(2) $125=5^3$ より，$\mathbf{5}$　**(答)**

(3) $-243=(-3)^5$ より，$\mathbf{-3}$　**(答)**

(4) $729=(27)^2$ より，$\pm\mathbf{27}$　**(答)**

(5) $729=9^3$ より，$\mathbf{9}$　**(答)**

【2】(1) $\sqrt[3]{4}\sqrt[3]{6}=\sqrt[3]{24}=\sqrt[3]{2^3\times 3}=\mathbf{2\sqrt[3]{3}}$　**(答)**

(2) $\dfrac{\sqrt[3]{162}}{\sqrt[3]{6}}=\sqrt[3]{\dfrac{162}{6}}=\sqrt[3]{27}=\sqrt[3]{3^3}=3^1=\mathbf{3}$　**(答)**

(3) $1024=2^{10}$ だから，

$$(\text{与式})=\left\{(2^{10})^{\frac{1}{2}}\right\}^{\frac{1}{5}}=(2^{10})^{\frac{1}{10}}=\mathbf{2}$$　**(答)**

(4) $\left(\dfrac{49}{64}\right)^{-1.5}=\left\{\left(\dfrac{7}{8}\right)^2\right\}^{-\frac{3}{2}}=\left(\dfrac{7}{8}\right)^{-3}=\mathbf{\dfrac{512}{343}}$　**(答)**

(5)
$$\begin{aligned}4^{-\frac{3}{2}}\times 27^{\frac{1}{3}}\div\sqrt{16^{-3}}&=\left(2^2\right)^{-\frac{3}{2}}\times\left(3^3\right)^{\frac{1}{3}}\div\left\{\left(2^4\right)^{-3}\right\}^{\frac{1}{2}}\\&=2^{-3}\times 3^1\div 2^{-6}=2^{-3-(-6)}\times 3^1\\&=\mathbf{24}\end{aligned}$$　**(答)**

(6) $\left(\sqrt[4]{5^3}\times\sqrt{27}\right)^{\frac{4}{3}}=\left(5^{\frac{3}{4}}\times\left(3^3\right)^{\frac{1}{2}}\right)^{\frac{4}{3}}=5^{\frac{3}{4}\times\frac{4}{3}}\times 3^{\frac{3}{2}\times\frac{4}{3}}=5\times 3^2=\mathbf{45}$　**(答)**

(7)
$$\begin{aligned}&2\sqrt{3}\times\sqrt[3]{24}\div\sqrt{6}\div\sqrt[6]{72}\\&=\left(2^1\times 3^{\frac{1}{2}}\right)\times\left(2^3\times 3\right)^{\frac{1}{3}}\times(2\times 3)^{-\frac{1}{2}}\times\left(2^3\times 3^2\right)^{-\frac{1}{6}}\\&=\left(2^1\times 3^{\frac{1}{2}}\right)\times\left(2^1\times 3^{\frac{1}{3}}\right)\times\left(2^{-\frac{1}{2}}\times 3^{-\frac{1}{2}}\right)\times\left(2^{-\frac{1}{2}}\times 3^{-\frac{1}{3}}\right)\\&=2^{1+1-\frac{1}{2}-\frac{1}{2}}\times 3^{\frac{1}{2}+\frac{1}{3}-\frac{1}{2}-\frac{1}{3}}=2^1\times 3^0=\mathbf{2}\end{aligned}$$　**(答)**

(8)
$$\begin{aligned}&\sqrt[3]{-54}\div\sqrt[3]{\sqrt[2]{128}}\div\sqrt[3]{-4}\\&=-\sqrt[3]{54}\times\frac{1}{\sqrt[6]{128}}\times\frac{1}{-\sqrt[3]{4}}=\sqrt[3]{\frac{54}{4}}\times\frac{1}{\sqrt[6]{128}}=\sqrt[3]{\frac{27}{2}}\times\frac{1}{\sqrt[6]{128}}\\&=\frac{\sqrt[3]{3^3}}{\sqrt[3]{2}}\times\frac{1}{\sqrt[6]{2^7}}=3\times 2^{-\frac{1}{3}}\times 2^{-\frac{7}{6}}=3\times 2^{-\frac{1}{3}-\frac{7}{6}}=3\times 2^{-\frac{3}{2}}\\&=\frac{3}{2\sqrt{2}}=\mathbf{\frac{3\sqrt{2}}{4}}\end{aligned}$$　**(答)**

(9) $81=3^4$ だから,

$$(\text{与式})=(3^4)^{-\frac{3}{4}}=3^{-3}=\mathbf{\frac{1}{27}} \qquad \textbf{(答)}$$

(10)
$$\begin{aligned}&(\sqrt[4]{5}-\sqrt[4]{3})(\sqrt[4]{5}+\sqrt[4]{3})(\sqrt{5}+\sqrt{3})\\ =&(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})\\ =&5-3=\mathbf{2} \qquad \textbf{(答)}\end{aligned}$$

(11)
$$\begin{aligned}&\left(5^{\frac{1}{2}}+5^{-\frac{1}{2}}\right)\left(5^{\frac{1}{2}}-5^{-\frac{1}{2}}\right)=\left(5^{\frac{1}{2}}\right)^2-\left(5^{-\frac{1}{2}}\right)^2\\ =&5^1-5^{-1}=5-\frac{1}{5}=\mathbf{\frac{24}{5}} \qquad \textbf{(答)}\end{aligned}$$

(12) $(\text{与式})=(\sqrt[3]{4})^3-(\sqrt[3]{3})^3=4-3=\mathbf{1}$ 　　**(答)**

【3】(1)
$$\begin{aligned}(8\sqrt{x})^{\frac{2}{3}}\times\sqrt[3]{x^2}&=\left(2^3\times x^{\frac{1}{2}}\right)^{\frac{2}{3}}\times x^{\frac{2}{3}}=(2^3)^{\frac{2}{3}}\times\left(x^{\frac{1}{2}}\right)^{\frac{2}{3}}\times x^{\frac{2}{3}}\\ &=2^{3\times\frac{2}{3}}\times x^{\frac{1}{2}\times\frac{2}{3}+\frac{2}{3}}=2^2\times x^1=\boldsymbol{4x} \qquad \textbf{(答)}\end{aligned}$$

(2)
$$\begin{aligned}\left(64x^3y^{-9}\right)^{\frac{1}{3}}&=\left(2^6\times x^3\times y^{-9}\right)^{\frac{1}{3}}\\ &=\left(2^6\right)^{\frac{1}{3}}\times\left(x^3\right)^{\frac{1}{3}}\times\left(y^{-9}\right)^{\frac{1}{3}}\\ &=2^2\times x^1\times y^{-3}=\boldsymbol{\frac{4x}{y^3}} \qquad \textbf{(答)}\end{aligned}$$

(3) $(\text{与式})=x^{\frac{4}{3}-\frac{2}{3}+\frac{1}{3}}y^{-\frac{1}{2}+\frac{1}{3}+\frac{1}{6}}=x^1y^0=\boldsymbol{x}$ 　　**(答)**

(4)
$$\begin{aligned}\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)^2&=\left(x^{\frac{1}{2}}\right)^2+2x^{\frac{1}{2}}x^{-\frac{1}{2}}+\left(x^{-\frac{1}{2}}\right)^2=x^1+2x^0+x^{-1}\\ &=\boldsymbol{x+2+\frac{1}{x}} \qquad \textbf{(答)}\end{aligned}$$

<参考>

$$(\text{与式})=\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=\boldsymbol{x+2+\frac{1}{x}} \qquad \textbf{(答)}$$

(5) $(\text{与式})=(x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}+1)=\boldsymbol{x-1}$ 　　**(答)**

(6) $(x-y)\div\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right)=\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right)\div\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right)=\boldsymbol{\sqrt{x}+\sqrt{y}}$ 　**(答)**

(7)
$$\begin{aligned}&\left(x^{\frac{1}{2}}+x^{\frac{1}{4}}y^{\frac{1}{4}}+y^{\frac{1}{2}}\right)\left(x^{\frac{1}{2}}-x^{\frac{1}{4}}y^{\frac{1}{4}}+y^{\frac{1}{2}}\right)\\ =&\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}+x^{\frac{1}{4}}y^{\frac{1}{4}}\right)\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}-x^{\frac{1}{4}}y^{\frac{1}{4}}\right)\\ =&\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)^2-\left(x^{\frac{1}{4}}y^{\frac{1}{4}}\right)^2=x^1+2x^{\frac{1}{2}}y^{\frac{1}{2}}+y^1-x^{\frac{1}{2}}y^{\frac{1}{2}}\\ =&\boldsymbol{x+\sqrt{xy}+y} \qquad \textbf{(答)}\end{aligned}$$

(8)
$$\begin{aligned}&\left(x^{\frac{a}{3}}+y^{-\frac{a}{3}}\right)\left\{x^{\frac{2a}{3}}-\left(xy^{-1}\right)^{\frac{a}{3}}+y^{-\frac{2a}{3}}\right\}=\left(x^{\frac{a}{3}}\right)^3+\left(y^{-\frac{a}{3}}\right)^3=x^a+y^{-a}\\ =&\boldsymbol{x^a+\frac{1}{y^a}} \qquad \textbf{(答)}\end{aligned}$$

(9)　$\left(xy^{-3}z^{3}\right)^{\frac{1}{2}} \times \left(x^{7}y^{4}z^{2}\right)^{\frac{1}{3}} \times \left(x^{-5}yz\right)^{\frac{1}{6}} = x^{\frac{1}{2}+\frac{7}{3}-\frac{5}{6}}y^{-\frac{3}{2}+\frac{4}{3}+\frac{1}{6}}z^{\frac{3}{2}+\frac{2}{3}+\frac{1}{6}} = x^{2}y^{0}z^{\frac{7}{3}}$

$= \boldsymbol{x^{2}z^{\frac{7}{3}}}$　**(答)**

(10)　$\left(x^{\frac{a}{a-b}}\right)^{\frac{1}{c-a}}\left(x^{\frac{b}{b-c}}\right)^{\frac{1}{a-b}}\left(x^{\frac{c}{c-a}}\right)^{\frac{1}{b-c}} = x^{\frac{a(b-c)+b(c-a)+c(a-b)}{(a-b)(b-c)(c-a)}} = x^{0} = \mathbf{1}$　**(答)**

(11)　$(与式) = (x^{3}-y^{-3}) \div (x^{2}+xy^{-1}+y^{-2})$

$= (x-y^{-1})(x^{2}+xy^{-1}+y^{-2}) \div (x^{2}+xy^{-1}+y^{-2})$

$= \boldsymbol{x - \frac{1}{y}}$　**(答)**

【4】(1) $\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)^{2} = 9$ より，

$x + x^{-1} + 2 = 9$

よって，

$x + x^{-1} = \mathbf{7}$　**(答)**

(2)　$x^{\frac{3}{2}}+x^{-\frac{3}{2}} = \left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)\left\{\left(x^{\frac{1}{2}}\right)^{2} - x^{\frac{1}{2}}\cdot x^{-\frac{1}{2}} + \left(x^{-\frac{1}{2}}\right)^{2}\right\}$

$= \left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)\left(x+x^{-1}-1\right)$

$= 3 \times (7-1) = \mathbf{18}$　**(答)**

【5】$a^{x} - a^{-x} = 0$ のとき，両辺に a^{x} をかけて，

$a^{2x} - 1 = 0 \qquad \therefore \quad a^{2x} = 1$

となり，(題意に) 矛盾するので，

$a^{x} - a^{-x} \neq 0$

$$\frac{a^{3x}-a^{-3x}}{a^{x}-a^{-x}} = \frac{(a^{x}-a^{-x})\left\{(a^{x})^{2}+a^{x}a^{-x}+(a^{-x})^{2}\right\}}{a^{x}-a^{-x}} = a^{2x} + \left(a^{2x}\right)^{-1} + 1$$

$$= 4 + 4^{-1} + 1 = 4 + \frac{1}{4} + 1 = \mathbf{\frac{21}{4}}$$　**(答)**

＜別解＞

$a^{2x} = 4$ より，$(a^{x})^{2} = 4$.

$a^{x} > 0$ より，$a^{x} = 2$.

よって，

$$(求値式) = \frac{(a^{x})^{3}-(a^{x})^{-3}}{a^{x}-(a^{x})^{-1}} = \frac{2^{3}-2^{-3}}{2-2^{-1}} = \frac{8-\frac{1}{8}}{2-\frac{1}{2}} = \mathbf{\frac{21}{4}}$$　(答)

【6】 $x=\dfrac{1}{2}\left(5^{\frac{1}{n}}-5^{-\frac{1}{n}}\right)$ のとき，

$$1+x^2=1+\left(\frac{1}{2}\left(5^{\frac{1}{n}}-5^{-\frac{1}{n}}\right)\right)^2=\left(\frac{1}{2}\left(5^{\frac{1}{n}}+5^{-\frac{1}{n}}\right)\right)^2$$

であるから，

$$\begin{aligned}(\text{求値式})&=\left\{\frac{1}{2}\left(5^{\frac{1}{n}}-5^{-\frac{1}{n}}\right)+\sqrt{\left(\frac{1}{2}\left(5^{\frac{1}{n}}+5^{-\frac{1}{n}}\right)\right)^2}\right\}^n\\&=\left\{\frac{1}{2}\left(5^{\frac{1}{n}}-5^{-\frac{1}{n}}\right)+\frac{1}{2}\left(5^{\frac{1}{n}}+5^{-\frac{1}{n}}\right)\right\}^n\\&=(5^{\frac{1}{n}})^n=\mathbf{5}\qquad\textbf{(答)}\end{aligned}$$

【7】(1) $3^x-3^{-x}=3$ より，

$$\begin{aligned}(3^x-3^{-x})^2=3^2\quad&\iff\quad 3^{2x}-2\cdot 3^x\cdot 3^{-x}+3^{-2x}=9\\&\iff\quad 3^{2x}+3^{-2x}=11\end{aligned}$$

そして

$$(3^x+3^{-x})^2=3^{2x}+2\cdot 3^x\cdot 3^{-x}+3^{-2x}=11+2=13$$

であり，$3^x+3^{-x}>0$ だから，

$$3^x+3^{-x}=\boldsymbol{\sqrt{13}}\qquad\textbf{(答)}$$

また，

$$\begin{aligned}3^{3x}+3^{-3x}&=(3^x+3^{-x})(3^{2x}-3^x\cdot 3^{-x}+3^{-2x})\\&=\sqrt{13}\cdot(11-1)\\&=\boldsymbol{10\sqrt{13}}\qquad\textbf{(答)}\end{aligned}$$

(2)

$$\begin{aligned}a^2&=3^{10}+2\cdot 3^5\cdot 3^{-5}+3^{-10}=3^{10}+3^{-10}+2\\b^2&=3^{10}-2\cdot 3^5\cdot 3^{-5}+3^{-10}=3^{10}+3^{-10}-2\end{aligned}$$

だから，

$$a^2+b^2=2(3^{10}+3^{-10})$$

$$\therefore\quad 3^{10}+3^{-10}=\boldsymbol{\frac{a^2+b^2}{2}}\qquad\textbf{(答)}$$

【8】 $$\left(10\sqrt[3]{2}\right)^3 = 2000$$

であり,

$$12^3 = 1728, \quad 13^3 = 2197$$

だから,

$$12 < 10\sqrt[3]{2} < 13$$

そして，$10\sqrt[3]{2}$ が，12 と 13 のどちらに近いかを考えればよい．ここで,

$$(13 - 10\sqrt[3]{2}) - (10\sqrt[3]{2} - 12) = 25 - 20\sqrt[3]{2} = 5(5 - 4\sqrt[3]{2})$$

であり,

$$5^3 = 125, \quad (4\sqrt[3]{2})^3 = 128$$

だから,

$$5 - 4\sqrt[3]{2} < 0$$

よって,

$$13 - 10\sqrt[3]{2} < 10\sqrt[3]{2} - 12$$

だから，求める値は,

13　　(答)

＜別解＞

$$12 < 10\sqrt[3]{2} < 13$$

を求めたあと,

$$\begin{aligned} 12.5^3 &= \left(\frac{25}{2}\right)^3 \\ &= \frac{15625}{8} < 2000 = \left(10\sqrt[3]{2}\right)^3 \end{aligned}$$ より

$12 < 12.5 < 10\sqrt[3]{2} < 13$
としてもよい.

添削課題

【1】(1)

$$
\begin{aligned}
2^3 \times 2^0 \times (2^{-1})^2 &= 8 \times 1 \times \left(\frac{1}{2}\right)^2 \\
&= \mathbf{2} \qquad \text{(答)}
\end{aligned}
$$

(2)

$$
\begin{aligned}
a^4 \div a^{-2} \times a^0 &= a^4 \div \frac{1}{a^2} \times 1 \\
&= \boldsymbol{a^6} \qquad \text{(答)}
\end{aligned}
$$

(3)

$$
\begin{aligned}
x^2 \div \left(\frac{1}{x}\right)^{-2} &= x^2 \div \left(x^{-1}\right)^{-2} \\
&= x^2 \div x^2 \\
&= \mathbf{1} \qquad \text{(答)}
\end{aligned}
$$

(4)

$$
\begin{aligned}
36x^{-1}y^2 \div (-2x^3y^{-1})^2 &= 36x^{-1}y^2 \div 4x^6y^{-2} \\
&= \frac{36y^2}{x} \div \frac{4x^6}{y^2} \\
&= \boldsymbol{\frac{9y^4}{x^7}} \qquad \text{(答)}
\end{aligned}
$$

【2】(1) 2 乗して 25 になる実数であるから，$\boldsymbol{\pm 5}$ 　(答)

(2) $\sqrt{25} = \mathbf{5}$ 　(答)

(3) 3 乗して -27 になる実数であるから，$\boldsymbol{-3}$ 　(答)

(4) $\sqrt[3]{-27} = \boldsymbol{-3}$ 　(答)

(5) 4 乗して 16 になる実数であるから，$\boldsymbol{\pm 2}$ 　(答)

(6) $\sqrt[4]{16} = \mathbf{2}$ 　(答)

【3】(1)

$$
\begin{aligned}
\left(\sqrt[4]{3}\right)^8 &= \sqrt[4]{3^8} = \left(\sqrt[4]{3^2}\right)^4 \\
&= 3^2 \\
&= \mathbf{9} \qquad \text{(答)}
\end{aligned}
$$

(2)

$$\sqrt[3]{32} \div \sqrt[3]{4} = \sqrt[3]{\frac{32}{4}} = \sqrt[3]{8} = \sqrt[3]{2^3}$$
$$= \left(\sqrt[3]{2}\right)^3$$
$$= \mathbf{2} \qquad \text{(答)}$$

(3)

$$\left(\frac{16}{81}\right)^{-0.75} = \left(\frac{16}{81}\right)^{-\frac{3}{4}} = \left(\frac{81}{16}\right)^{\frac{3}{4}} = \left(\frac{3^4}{2^4}\right)^{\frac{3}{4}}$$
$$= \frac{3^3}{2^3}$$
$$= \frac{\mathbf{27}}{\mathbf{8}} \qquad \text{(答)}$$

(4)

$$4^{\frac{2}{3}} \div 40^{\frac{1}{3}} \times 50^{\frac{2}{3}} = \left(2^2\right)^{\frac{2}{3}} \times \left(2^3 \cdot 5\right)^{-\frac{1}{3}} \times \left(2 \cdot 5^2\right)^{\frac{2}{3}}$$
$$= 2^{2\times\frac{2}{3}+3\times\left(-\frac{1}{3}\right)+1\times\frac{2}{3}} \cdot 5^{-\frac{1}{3}+2\times\frac{2}{3}}$$
$$= 2^1 \cdot 5^1$$
$$= \mathbf{10} \qquad \text{(答)}$$

【4】(1)

$$\left(3^x + 3^{-x}\right)^2 = \left(3^2\right)^x + 2 \cdot 3^x \cdot 3^{-x} + \left(3^2\right)^{-x}$$
$$= 9^x + 2 \cdot 1 + 9^{-x}$$
$$= 10 + 2 = 12$$

$3^x + 3^{-x} > 0$ であるから

$$\therefore\ 3^x + 3^{-x} = \mathbf{2\sqrt{3}} \qquad \text{(答)}$$

(2)

$$\left(3^x - 3^{-x}\right)^2 = \left(3^2\right)^x - 2 \cdot 3^x \cdot 3^{-x} + \left(3^2\right)^{-x}$$
$$= 9^x - 2 \cdot 1 + 9^{-x}$$
$$= 10 - 2 = 8$$

ここで，$x > 0$ より，$3^x > 1$，$\dfrac{1}{3^x} < 1$ であるから，

$$3^x - 3^{-x} > 0$$

よって，

$$\therefore\ 3^x - 3^{-x} = \mathbf{2\sqrt{2}} \qquad \text{(答)}$$

(3)

$$27^x - 27^{-x} = \left(3^3\right)^x - \left(3^3\right)^{-x}$$
$$= \left(3^x\right)^3 - \left(3^{-x}\right)^3$$
$$= \left(3^x - 3^{-x}\right)\left(9^x + 3^x \cdot 3^{-x} + 9^{-x}\right)$$
$$= 2\sqrt{2} \cdot (10 + 1)$$
$$= \mathbf{22\sqrt{2}} \qquad \text{(答)}$$

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