

Z会東大進学教室

中 2 選抜東大・医学部数学

中 2 数学

中 2 東大数学



14章 平方根 (1)

問題

【1】 a の平方根は、2乗して a になる数で、 $a > 0$ のとき、 a の平方根は $\pm\sqrt{a}$ であり、特に、 $a = 0$ のとき、 a の平方根は 0 であることより、答えは以下ようになる。

- (1) ± 2 (2) $\pm\sqrt{2}$ (3) ± 5 (4) ± 11
 (5) $\pm\sqrt{17}$ (6) 0 (7) $\pm\frac{3}{4}$ (8) ± 0.1
 (9) ± 0.08 (10) なし

【2】 (1) $\sqrt{4} = \sqrt{2^2} = 2$

(2) $\sqrt{\frac{9}{49}} = \sqrt{\frac{3^2}{7^2}} = \frac{3}{7}$

(3) $-\sqrt{1\frac{13}{36}} = -\sqrt{\frac{49}{36}} = -\sqrt{\left(\frac{7}{6}\right)^2} = -\frac{7}{6}$

(4) $\sqrt{(-5)^2} = \sqrt{5^2} = 5$ (5) $\sqrt{\left(\frac{3}{2}\right)^2} = \frac{3}{2}$

(6) $\sqrt{\frac{3^2}{4}} = \sqrt{\left(\frac{3}{2}\right)^2} = \frac{3}{2}$ (7) $(-\sqrt{8})^2 = \sqrt{8^2} = 8$

(8) $-(\sqrt{6})^2 = -\sqrt{6^2} = -6$

【3】 (1) 6 の平方根は $\pm\sqrt{6}$ だから、誤り。 $3 \rightarrow \pm\sqrt{6}$

(2) 25 の平方根は ± 5 だから、誤り。 $5 \rightarrow \pm 5$

(3) $\sqrt{3} \times \sqrt{3} = \sqrt{3 \times 3} = \sqrt{3^2} = 3$ だから、誤り。 $2\sqrt{3} \rightarrow 3$

(4) $\sqrt{36} = \sqrt{6^2} = 6$ だから、誤り。 $\pm 6 \rightarrow 6$

(5) $(-\sqrt{2})^2 = \sqrt{2^2} = 2$ だから、誤り。 $-2 \rightarrow 2$

(6) $\sqrt{0.09} = \sqrt{0.3^2} = 0.3$ だから、正しい。 ○

(7) -6 は 2 乗すると 36 になるので、36 の平方根。 だから、正しい。 ○

(8) -2 は 2 乗すると 4 になるので、4 の平方根。 だから、誤り。 $-4 \rightarrow 4$

【4】 (1) $-5, 0, \sqrt{16}$ (2) $\sqrt{16}$

(3) $-5, -\sqrt{\frac{4}{9}}, -\frac{1}{3}, 0, 0.7, \sqrt{16}$ (4) $-\sqrt{17}, \frac{\sqrt{2}}{2}, \frac{\pi}{2}$

【5】 (1) $6 = \sqrt{36}$ より, $35 < 36$
よって, $\sqrt{35}$, **6**

(2) $\frac{1}{2} = \sqrt{\frac{1}{4}}$ より, $\frac{1}{5} < \frac{1}{4} < \frac{1}{3}$
よって, $\sqrt{\frac{1}{5}}$, $\frac{1}{2}$, $\sqrt{\frac{1}{3}}$

(3) $-5.1 = -\sqrt{26.01}$ より, $26 < 26.01$
ゆえに, $\sqrt{26} < \sqrt{26.01}$
すなわち, $-\sqrt{26.01} < -\sqrt{26}$
よって, **-5.1**, $-\sqrt{26}$

(4) $-\frac{1}{4} = -\sqrt{\frac{1}{16}}$, $-\sqrt{0.2} = -\sqrt{\frac{1}{5}}$ より, $\frac{1}{18} < \frac{1}{16} < \frac{1}{5}$
ゆえに, $\sqrt{\frac{1}{18}} < \sqrt{\frac{1}{16}} < \sqrt{\frac{1}{5}}$
すなわち, $-\sqrt{\frac{1}{5}} < -\sqrt{\frac{1}{16}} < -\sqrt{\frac{1}{18}}$
よって, $-\sqrt{0.2}$, $-\frac{1}{4}$, $-\sqrt{\frac{1}{18}}$

(5) $2.5 = \sqrt{6.25}$, $\sqrt{3^2} = \sqrt{9}$ より, $6.25 < 8 < 9$
よって, **2.5**, $\sqrt{8}$, $\sqrt{3^2}$

(6) $-1.5 = -\sqrt{2.25}$, $-3 = -\sqrt{9}$ より, $2.25 < 3 < 9$
ゆえに, $\sqrt{2.25} < \sqrt{3} < \sqrt{9}$
すなわち, $-\sqrt{9} < -\sqrt{3} < -\sqrt{2.25}$
よって, **-3**, $-\sqrt{3}$, **-1.5**

【6】 (1) ① $\sqrt{9} < \sqrt{a} < \sqrt{16}$ より, $9 < a < 16$
 a は最小の整数だから, **$a = 10$**

② $\sqrt{53.29} < \sqrt{a} < \sqrt{81}$ より, $53.29 < a < 81$
 a は最小の整数だから, **$a = 54$**

③ $-\sqrt{25} < -\sqrt{a} < -\sqrt{4}$
辺々に -1 をかけて, $\sqrt{4} < \sqrt{a} < \sqrt{25}$ より, $4 < a < 25$
 a は最小の整数だから, **$a = 5$**

④ $-\sqrt{36} < -\sqrt{a} < -\sqrt{22.09}$
辺々に -1 をかけて, $\sqrt{22.09} < \sqrt{a} < \sqrt{36}$ より, $22.09 < a < 36$
 a は最小の整数だから, **$a = 23$**

(2) ① $\sqrt{4} < \sqrt{k} < \sqrt{9}$ より, $4 < k < 9$

k は自然数だから, $k = 5, 6, 7, 8$

② $\sqrt{\frac{25}{4}} < \sqrt{6k} < \sqrt{\frac{81}{4}}$
 $1\frac{1}{24} = \frac{25}{24} < k < \frac{27}{8} = 3\frac{3}{8}$
 よって, $k = 2, 3$

③ $\frac{1}{3} < \frac{1}{\sqrt{k}} < \frac{1}{2}$ $\sqrt{\frac{1}{9}} < \sqrt{\frac{1}{k}} < \sqrt{\frac{1}{4}}$ より, $\frac{1}{9} < \frac{1}{k} < \frac{1}{4}$
 辺々を逆数にして, $4 < k < 9$
 k は自然数だから, $k = 5, 6, 7, 8$

【7】 (1) $\sqrt{18n} = \sqrt{2 \times 3^2 \times n} = 3\sqrt{2 \times n}$ より, $n = 2$ とすればよい. ($\sqrt{18n} = 3\sqrt{2^2} = 6$)

(2) $\sqrt{120n} = \sqrt{2^3 \times 3 \times 5 \times n} = 2\sqrt{2 \times 3 \times 5 \times n}$ より, $n = 2 \times 3 \times 5 = 30$ とすればよい. ($\sqrt{120n} = 2\sqrt{2^2 \times 3^2 \times 5^2} = 60$)

(3) $\sqrt{\frac{2520}{n}} = \sqrt{\frac{2^3 \times 3^2 \times 5 \times 7}{n}} = 6\sqrt{\frac{2 \times 5 \times 7}{n}}$ より, $n = 2 \times 5 \times 7 = 70$ とすればよい. ($\sqrt{\frac{2520}{n}} = \sqrt{\frac{2520}{70}} = \sqrt{36} = 6$)

(4) $\sqrt{\frac{27}{140}n} = \sqrt{\frac{3^3}{2^2 \times 5 \times 7}n} = \frac{3}{2}\sqrt{\frac{3n}{5 \times 7}}$ より, $n = 3 \times 5 \times 7 = 105$ とすればよい. ($\sqrt{\frac{27}{140} \times 105} = \frac{3}{2}\sqrt{\frac{3}{5 \times 7} \times 3 \times 5 \times 7} = \frac{3}{2} \times 3 = \frac{9}{2}$)

【8】 (1) $\sqrt{3} \times \sqrt{7} = \sqrt{21}$ (2) $-\sqrt{5} \times \sqrt{11} = -\sqrt{55}$

(3) $2\sqrt{3} \times \sqrt{5} = 2\sqrt{3 \times 5} = 2\sqrt{15}$

(4) $-3\sqrt{7} \times 2\sqrt{2} = -3 \times \sqrt{7} \times 2 \times \sqrt{2} = -6 \times \sqrt{7 \times 2} = -6\sqrt{14}$

(5) $\sqrt{35} \div \sqrt{7} = \sqrt{\frac{35}{7}} = \sqrt{5}$ (6) $-\sqrt{39} \div \sqrt{3} = -\sqrt{\frac{39}{3}} = -\sqrt{13}$

(7) $2\sqrt{10} \div \sqrt{2} = 2 \times \sqrt{\frac{10}{2}} = 2\sqrt{5}$ (8) $2\sqrt{42} \div \sqrt{6} = 2 \times \sqrt{\frac{42}{6}} = 2\sqrt{7}$

(9) $-3\sqrt{22} \div (-\sqrt{11}) = 3\sqrt{\frac{22}{11}} = 3\sqrt{2}$

(10) $4\sqrt{91} \div (-3\sqrt{7}) = -\frac{4}{3}\sqrt{\frac{91}{7}} = -\frac{4\sqrt{13}}{3}$

【9】 (1) $2\sqrt{3} = \sqrt{4 \times 3} = \sqrt{12}$

(3) $-5\sqrt{2} = -\sqrt{5^2 \times 2} = -\sqrt{50}$

(5) $\sqrt{56} = \sqrt{2^2 \times 14} = 2\sqrt{14}$

(7) $\sqrt{108} = \sqrt{2^2 \times 3^3}$
 $= 2 \times 3 \times \sqrt{3}$
 $= 6\sqrt{3}$

(9) $2\sqrt{135} = 2\sqrt{3^3 \times 5}$
 $= 2 \times 3 \times \sqrt{3 \times 5}$
 $= 6\sqrt{15}$

(2) $4\sqrt{7} = \sqrt{4^2 \times 7} = \sqrt{112}$

(4) $\frac{2}{3}\sqrt{7} = \sqrt{\left(\frac{2}{3}\right)^2 \times 7} = \sqrt{\frac{28}{9}}$

(6) $\sqrt{125} = \sqrt{5^2 \times 5} = 5\sqrt{5}$

(8) $-\sqrt{4200} = -\sqrt{2^3 \times 3 \times 5^2 \times 7}$
 $= -2 \times 5 \times \sqrt{2 \times 3 \times 7}$
 $= -10\sqrt{42}$

(10) $-3\sqrt{98} = -3\sqrt{2 \times 7^2}$
 $= -3 \times 7\sqrt{2}$
 $= -21\sqrt{2}$

【10】 (1) $\sqrt{300} = 10\sqrt{3} = 10 \times 1.73 = 17.3$

(3) $\sqrt{5} \times \sqrt{10} = 5\sqrt{2}$
 $= 5 \times 1.41$
 $= 7.05$

(2) $\sqrt{8} = 2\sqrt{2} = 2 \times 1.41 = 2.82$

(4) $\sqrt{2} \times \sqrt{12} = \sqrt{2 \times 2^2 \times 3}$
 $= 2\sqrt{6}$
 $= 2 \times \sqrt{2} \times \sqrt{3}$
 $= 2 \times 1.41 \times 1.73$
 $= 4.8786$
 $\div 4.88$

【11】 (1) $\sqrt{3} \times \sqrt{15} = \sqrt{45} = 3\sqrt{5}$

(3) $\sqrt{3} \times (-\sqrt{6}) = -\sqrt{18} = -3\sqrt{2}$

(5) $\sqrt{2} \times 2\sqrt{6} = 2 \times \sqrt{2 \times 6}$
 $= 2\sqrt{12}$
 $= 2 \times 2\sqrt{3}$
 $= 4\sqrt{3}$

(7) $-3\sqrt{14} \times (-2\sqrt{28})$
 $= (-3) \times (-2) \times \sqrt{14 \times 14 \times 2}$
 $= 6 \times 14\sqrt{2}$
 $= 84\sqrt{2}$

(2) $-\sqrt{14} \times \sqrt{21} = -\sqrt{2 \times 7 \times 7 \times 3}$
 $= -7\sqrt{6}$

(4) $-\sqrt{30} \times (-\sqrt{15}) = +\sqrt{15 \times 2 \times 15}$
 $= 15\sqrt{2}$

(6) $\sqrt{6} \times 3\sqrt{3} = 3 \times \sqrt{6 \times 3}$
 $= 3\sqrt{18}$
 $= 3 \times 3\sqrt{2}$
 $= 9\sqrt{2}$

(8) $-3\sqrt{22} \times 2\sqrt{11}$
 $= (-3) \times 2 \times \sqrt{2 \times 11 \times 11}$
 $= -6 \times 11\sqrt{2}$
 $= -66\sqrt{2}$

$$(9) \sqrt{56} \div \sqrt{7} = \sqrt{\frac{56}{7}} = \sqrt{8} = 2\sqrt{2} \quad (10) \sqrt{180} \div \sqrt{15} = \sqrt{\frac{180}{15}} = \sqrt{12} = 2\sqrt{3}$$

$$(11) 3\sqrt{18} \div \sqrt{6} = 3 \times \sqrt{\frac{18}{6}} = 3\sqrt{3}$$

$$(12) \sqrt{28} \div 3\sqrt{7} = \frac{1}{3} \times \sqrt{\frac{28}{7}} = \frac{1}{3} \times \sqrt{4} = \frac{2}{3}$$

$$(13) 2\sqrt{84} \div 4\sqrt{6} = \frac{2}{4} \sqrt{\frac{84}{6}} = \frac{\sqrt{14}}{2} \quad (14) 12\sqrt{15} \div 9\sqrt{5} = \frac{12}{9} \sqrt{\frac{15}{5}} = \frac{4\sqrt{3}}{3}$$

[12] (1)
$$\begin{aligned} \sqrt{2} \times \sqrt{3} \div \sqrt{6} &= \sqrt{\frac{2 \times 3}{6}} \\ &= \sqrt{1} \\ &= 1 \end{aligned}$$

(2)
$$\begin{aligned} \sqrt{75} \div \sqrt{5} \div \sqrt{3} &= \sqrt{\frac{75}{5 \times 3}} \\ &= \sqrt{5} \end{aligned}$$

(3)
$$\begin{aligned} \sqrt{6} \div (-\sqrt{3}) \times \sqrt{2} \\ &= -\sqrt{\frac{6 \times 2}{3}} \\ &= -2 \end{aligned}$$

(4)
$$\begin{aligned} (-\sqrt{8}) \times \sqrt{18} \div (-\sqrt{12}) \\ &= \sqrt{\frac{8 \times 18}{12}} \\ &= \sqrt{12} \\ &= 2\sqrt{3} \end{aligned}$$

(5)
$$\begin{aligned} \sqrt{180} \div (-2\sqrt{5}) \div \sqrt{3} \\ &= \sqrt{180} \times \left(-\sqrt{\frac{1}{20}}\right) \times \sqrt{\frac{1}{3}} \\ &= -\sqrt{\frac{180}{20 \times 3}} \\ &= -\sqrt{3} \end{aligned}$$

(6)
$$\begin{aligned} -\sqrt{3} \times \sqrt{6} \div (-\sqrt{8}) \\ &= \sqrt{\frac{3 \times 6}{8}} \\ &= \frac{3}{2} \end{aligned}$$

(7)
$$\begin{aligned} 4\sqrt{18} \div (-2\sqrt{2}) \times \frac{\sqrt{5}}{2} \\ &= 12\sqrt{2} \times \left(-\frac{1}{2\sqrt{2}}\right) \times \frac{\sqrt{5}}{2} \\ &= -\frac{12}{2 \times 2} \times \sqrt{\frac{2 \times 5}{2}} \\ &= -3\sqrt{5} \end{aligned}$$

(8)
$$\begin{aligned} (-3\sqrt{24}) \div 2\sqrt{2} \times (-\sqrt{27}) \\ &= -6\sqrt{6} \times \frac{1}{2\sqrt{2}} \times (-3\sqrt{3}) \\ &= \frac{6 \times 3}{2} \times \sqrt{\frac{6 \times 3}{2}} \\ &= 9 \times 3 \\ &= 27 \end{aligned}$$

$$\begin{aligned}
 (9) \quad & \sqrt{72} \div \sqrt{\frac{5}{6}} \times \sqrt{15} \\
 &= \sqrt{\frac{72 \times 6 \times 15}{5}} \\
 &= \sqrt{2^3 \times 3^2 \times 2 \times 3 \times 3} \\
 &= 4 \times 9 \\
 &= \mathbf{36}
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & (-\sqrt{2}) \div \sqrt{\frac{20}{3}} \times \sqrt{10} \\
 &= (-\sqrt{2}) \times \sqrt{\frac{3}{20}} \times \sqrt{10} \\
 &= -\sqrt{\frac{2 \times 3 \times 10}{20}} \\
 &= -\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 (11) \quad & \sqrt{\frac{3}{4}} \times \sqrt{\frac{8}{27}} \div (-\sqrt{2}) \\
 &= -\sqrt{\frac{3 \times 8 \times 1}{4 \times 27 \times 2}} \\
 &= -\frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 (12) \quad & \sqrt{\frac{8}{7}} \div \sqrt{\frac{2}{21}} \times \sqrt{3} \\
 &= \sqrt{\frac{8 \times 21 \times 3}{7 \times 2}} \\
 &= \sqrt{36} \\
 &= \mathbf{6}
 \end{aligned}$$

【13】 (1) $3.8^2 = 14.44$, $3.9^2 = 15.21$ より, 3.8 と 3.9 の間にあることがわかる.

$3.85^2 = 14.8225$ より, 3.9 により近いことがわかる. よって, ⑤

(2) $3.87^2 = 14.9769$, $3.88^2 = 15.0544$ より, 3.87 と 3.88 の間にあることがわかるので, 小数点以下第 2 位は **7**

【14】 (1) $19 \div 100 = \mathbf{0.19}$

(2) $3 \div 8 = \mathbf{0.375}$

(3) $5 \div 9 = 0.5555 \dots = \mathbf{0.\dot{5}}$

(4) $7 \div 33 = 0.212121 \dots = \mathbf{0.\dot{2}1}$

(5) $0.32 = \frac{32}{100} = \frac{\mathbf{8}}{\mathbf{25}}$

(6) $x = 0.\dot{7} \dots \textcircled{1}$

とおく. ①の両辺を 10 倍して

$$10x = 7.\dot{7} \dots \textcircled{2}$$

② - ① より,

$$9x = 7$$

$$x = \frac{\mathbf{7}}{\mathbf{9}}$$

(7) $x = 2.\dot{5}4 \dots \textcircled{1}$

とおく. ①の両辺を 100 倍して

$$100x = 254.\dot{5}4 \dots \textcircled{2}$$

② - ① より,

$$99x = 252$$

$$x = \frac{252}{99} = \frac{\mathbf{28}}{\mathbf{11}}$$

(8) $x = 0.1\dot{2}0\dot{6} \dots \textcircled{1}$

とおく. ①の両辺を 1000 倍して

$$1000x = 120.6\dot{2}0\dot{6} \dots \textcircled{2}$$

② - ① より,

$$999x = 120.5$$

$$x = \frac{1205}{9990} = \frac{\mathbf{241}}{\mathbf{1998}}$$

【15】 (1) $\sqrt{n^2 - 141}$ が整数ということは根号の中が平方数になるということなので, $n^2 -$

$141 = m^2$ とおくことができる.

$$\therefore n^2 - m^2 = 141$$

$$(n - m)(n + m) = 1 \cdot 141 = 3 \cdot 47$$

$n - m < n + m$ であり、共に整数なので、条件を満たす $n - m, n + m$ の組み合わせは

$$\begin{cases} n - m = 1 \\ n + m = 141 \end{cases} \quad \text{と} \quad \begin{cases} n - m = 3 \\ n + m = 47 \end{cases}$$

の2組である、それぞれ解くと

$$(n, m) = (71, 70), (25, 22)$$

これらは条件を満たすので、

$$\mathbf{n = 71, 25}$$

$$(2) \quad x + 100 = n^2 \dots \textcircled{1}, \quad x + 168 = m^2 \dots \textcircled{2} \quad (m, n \text{ は整数})$$

と表せばよい。

$$\textcircled{2} - \textcircled{1} \text{ より}$$

$$m^2 - n^2 = 168 - 100 = 68$$

$$\therefore (m - n)(m + n) = 68 = 1 \cdot 68 = 2 \cdot 34 = 4 \cdot 17$$

$m - n < m + n$ で、いずれも整数なので

$$\begin{cases} m - n = 1 \\ m + n = 68 \end{cases} \quad \begin{cases} m - n = 2 \\ m + n = 34 \end{cases} \quad \begin{cases} m - n = 4 \\ m + n = 17 \end{cases}$$

これらを解くと、

$$(m, n) = \left(\frac{69}{2}, \frac{67}{2}\right), (18, 16), \left(\frac{21}{2}, \frac{13}{2}\right)$$

しかし、 m, n は整数なので条件を満たすのは、

$$(m, n) = (18, 16)$$

このとき、

$$\begin{aligned} x &= n^2 - 100 \\ &= 256 - 100 = \mathbf{156} \end{aligned}$$

$$\mathbf{【16】(1) \quad b\sqrt{2} = -a}$$

$b \neq 0$ とし、両辺を b で割ると

$$\sqrt{2} = -\frac{a}{b}$$

このとき、 a, b は有理数なので、右辺は有理数。ところが、左辺は無理数であり、無理数と有理数は等しくならないので矛盾が生じている。

$\therefore b = 0$ でなければならない

$b = 0$ のとき

$$a + 0 \times \sqrt{2} = 0 \quad \therefore a = 0$$

よって、 $a = b = 0$ (証明終)

$$(2) \quad a + b\sqrt{2} = 4 + 3\sqrt{2}$$

$$(a - 4) + (b - 3)\sqrt{2} = 0$$

ここで、 $a - 4, b - 3$ は有理数であるから、(1) より

$$a - 4 = b - 3 = 0$$

$$\therefore \mathbf{a = 4, b = 3}$$

【5】 (1) $\sqrt{2} \times \sqrt{6} = \sqrt{2 \times 6} = \sqrt{2^2 \times 3} = 2\sqrt{3}$
 (2) $2\sqrt{3} \times (-3\sqrt{2}) = -2 \times 3 \times \sqrt{3 \times 2} = -6\sqrt{6}$
 (3) $\sqrt{12} \div \sqrt{2} = \sqrt{12} \times \frac{1}{\sqrt{2}} = \sqrt{\frac{12}{2}} = \sqrt{6}$
 (4) $\sqrt{\frac{3}{2}} \div \frac{\sqrt{6}}{2} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{2}{\sqrt{6}} = \frac{\sqrt{3} \times 2}{\sqrt{2} \times \sqrt{2} \times 3} = \frac{2\sqrt{3}}{2\sqrt{3}} = 1$
 (5) $\sqrt{40} \div \frac{3\sqrt{5}}{\sqrt{2}} = \sqrt{40} \times \frac{\sqrt{2}}{3\sqrt{5}} = \frac{\sqrt{2^2 \times 2 \times 5} \times \sqrt{2}}{3\sqrt{5}} = \frac{4\sqrt{5}}{3\sqrt{5}} = \frac{4}{3}$
 (6) $\frac{2}{\sqrt{10}} \div \frac{\sqrt{35}}{5} \times \frac{\sqrt{14}}{4} = \frac{2}{\sqrt{10}} \times \frac{5}{\sqrt{35}} \times \frac{\sqrt{14}}{4}$

$$= \frac{2 \times 5 \times \sqrt{2 \times 7}}{\sqrt{2 \times 5} \times \sqrt{5 \times 7} \times 4}$$

$$= \frac{2 \times 5 \sqrt{2 \times 7}}{4 \times 5 \sqrt{2 \times 7}}$$

$$= \frac{1}{2}$$

【6】 (1) $\boxed{\text{ア}} = 196, \boxed{\text{イ}} = 225$
 $\boxed{\text{ウ}} = 14, \boxed{\text{エ}} = 15$

(2) $2400 < 2401 = 49^2$ を利用すると,

$$\therefore \sqrt{2400} < 49$$

$$20\sqrt{6} < 49$$

$$\sqrt{6} < \frac{49}{20} = 2.45$$

一方, $244^2 = 59536 < 60000$ より, $244 < \sqrt{60000} = 100\sqrt{6}$

$$\therefore 2.44 < \sqrt{6}$$

以上より, $2.44 < \sqrt{6} < 2.45$

よって, $\sqrt{6}$ の小数第 2 位までは 2.44 であることが示された.

小テスト

- 【1】 (1) 相似比が $8 : 12 = 2 : 3$ なので, 体積比は $2^3 : 3^3 = 8 : 27$
(2) 表面積比は $3^2 : 4^2 = 9 : 16$ なので, 体積比は $3^3 : 4^3 = 27 : 64$
(3) 体積比は $10 \text{ 万} = 100000 = 10^5$ の 3 乗

よって

$$\begin{aligned} 3 \times \left(\frac{1}{100}\right)^3 (\text{m}^3) \times (10^5)^3 &= 3 \times \left(\frac{10^5}{10^2}\right)^3 = 3 \times (10^3)^3 = 3 \times 10^9 \\ &= 3000000000 = \mathbf{30 \text{ 億} (\text{m}^3)} \end{aligned}$$

15章 平方根 (2)

問題

- 【1】** (1) $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ (2) $\sqrt{7} \times (-\sqrt{11}) = -\sqrt{77}$
 (3) $2\sqrt{3} \times \sqrt{7} = 2\sqrt{21}$ (4) $-3\sqrt{2} \times 4\sqrt{13} = -12\sqrt{26}$
 (5) $\sqrt{42} \div \sqrt{3} = \sqrt{14}$ (6) $-\sqrt{39} \div \sqrt{13} = -\sqrt{3}$
 (7) $12\sqrt{22} \div \sqrt{2} = 12\sqrt{11}$ (8) $-10\sqrt{15} \div 5\sqrt{3} = -2\sqrt{5}$
 (9) $2\sqrt{6} \times 3\sqrt{3} = 6\sqrt{18} = 18\sqrt{2}$ (10) $-4\sqrt{5} \times 2\sqrt{15} = -8\sqrt{75} = -40\sqrt{3}$
- 【2】** (1) $3\sqrt{2} + 7\sqrt{2} = (3 + 7)\sqrt{2}$
 $= 10\sqrt{2}$ (2) $5\sqrt{3} - 9\sqrt{3} = (5 - 9)\sqrt{3}$
 $= -4\sqrt{3}$
- (3) $-6\sqrt{7} - 4\sqrt{7} = (-6 - 4)\sqrt{7}$
 $= -10\sqrt{7}$ (4) $\sqrt{3} - 3\sqrt{3} + 4\sqrt{3} = (1 - 3 + 4)\sqrt{3}$
 $= 2\sqrt{3}$
- (5) $4\sqrt{2} - \sqrt{3} + \sqrt{2} - 7\sqrt{3}$
 $= (4 + 1)\sqrt{2} + (-1 - 7)\sqrt{3}$
 $= 5\sqrt{2} - 8\sqrt{3}$ (6) $\sqrt{10} - \sqrt{5} - 9\sqrt{10} - \sqrt{5}$
 $= (1 - 9)\sqrt{10} + (-1 - 1)\sqrt{5}$
 $= -8\sqrt{10} - 2\sqrt{5}$
- (7) $12 + 3\sqrt{6} - \sqrt{3} - 3\sqrt{6} - 4\sqrt{3}$
 $= 12 + (3 - 3)\sqrt{6} + (-1 - 4)\sqrt{3}$
 $= 12 - 5\sqrt{3}$ (8) $8\sqrt{13} - 2\sqrt{2} + 13 - 3\sqrt{2} + \sqrt{13}$
 $= 13 + (-2 - 3)\sqrt{2} + (8 + 1)\sqrt{13}$
 $= 13 - 5\sqrt{2} + 9\sqrt{13}$
- 【3】** (1) $\sqrt{12} + \sqrt{3} = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$ (2) $-2\sqrt{2} + \sqrt{8} = -2\sqrt{2} + 2\sqrt{2} = 0$
 (3) $\sqrt{45} + 2\sqrt{5} = 3\sqrt{5} + 2\sqrt{5}$
 $= 5\sqrt{5}$ (4) $\sqrt{75} + \sqrt{12} = 5\sqrt{3} + 2\sqrt{3}$
 $= 7\sqrt{3}$
- (5) $3\sqrt{8} - \sqrt{50} = 6\sqrt{2} - 5\sqrt{2}$
 $= \sqrt{2}$ (6) $3\sqrt{28} + 4\sqrt{63} = 6\sqrt{7} + 12\sqrt{7}$
 $= 18\sqrt{7}$
- (7) $\sqrt{75} + \sqrt{108} + \sqrt{48}$
 $= 5\sqrt{3} + 6\sqrt{3} + 4\sqrt{3}$
 $= 15\sqrt{3}$ (8) $3\sqrt{6} + \sqrt{216} - 2\sqrt{24}$
 $= 3\sqrt{6} + 6\sqrt{6} - 4\sqrt{6}$
 $= 5\sqrt{6}$

$$\begin{aligned}
 (9) \quad & -5\sqrt{18} + 3\sqrt{8} - 2\sqrt{32} \\
 & = -15\sqrt{2} + 6\sqrt{2} - 8\sqrt{2} \\
 & = -17\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & \sqrt{32} - 3\sqrt{6} - 2\sqrt{18} + \sqrt{24} \\
 & = 4\sqrt{2} - 3\sqrt{6} - 6\sqrt{2} + 2\sqrt{6} \\
 & = -2\sqrt{2} - \sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{【4】 (1) ①} \quad & \frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\
 & = \frac{\sqrt{2}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{②} \quad & \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\
 & = \frac{5\sqrt{2}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{③} \quad & \frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{3} \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} \\
 & = \frac{\sqrt{21}}{7}
 \end{aligned}$$

$$\begin{aligned}
 \text{④} \quad & \frac{5}{\sqrt{15}} = \frac{5 \times \sqrt{15}}{\sqrt{15} \times \sqrt{15}} \\
 & = \frac{5\sqrt{15}}{15} \\
 & = \frac{\sqrt{15}}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{⑤} \quad & \frac{11}{3\sqrt{7}} = \frac{11\sqrt{7}}{3\sqrt{7} \times \sqrt{7}} \\
 & = \frac{11\sqrt{7}}{21}
 \end{aligned}$$

$$\begin{aligned}
 \text{⑥} \quad & \frac{3}{2\sqrt{3}} = \frac{3\sqrt{3}}{2\sqrt{3} \times \sqrt{3}} \\
 & = \frac{3\sqrt{3}}{2 \times 3} \\
 & = \frac{\sqrt{3}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{⑦} \quad & \frac{5\sqrt{2}}{\sqrt{45}} = \frac{5\sqrt{2}}{3\sqrt{5}} \\
 & = \frac{5\sqrt{2} \times \sqrt{5}}{3\sqrt{5} \times \sqrt{5}} \\
 & = \frac{5\sqrt{10}}{3 \times 5} \\
 & = \frac{\sqrt{10}}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{⑧} \quad & \sqrt{\frac{5}{6}} = \frac{\sqrt{5}}{\sqrt{6}} \\
 & = \frac{\sqrt{5} \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} \\
 & = \frac{\sqrt{30}}{6}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{9} \quad \sqrt{\frac{42}{63}} &= \sqrt{\frac{6}{9}} \\
 &= \sqrt{\frac{2}{3}} \\
 &= \frac{\sqrt{2}}{\sqrt{3}} \\
 &= \frac{\sqrt{2} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\
 &= \frac{\sqrt{6}}{3}
 \end{aligned}$$

$$\begin{aligned}
 (2) \textcircled{1} \quad \sqrt{10} \div \sqrt{15} \\
 &= \sqrt{\frac{10}{15}} \\
 &= \frac{\sqrt{2}}{\sqrt{3}} \\
 &= \frac{\sqrt{6}}{3}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad \frac{\sqrt{2}}{\sqrt{7}} \div \sqrt{21} \\
 &= \frac{\sqrt{2}}{\sqrt{7}} \times \sqrt{\frac{1}{21}} = \sqrt{\frac{2 \times 1}{7 \times 21}} \\
 &= \sqrt{\frac{2}{147}} = \frac{\sqrt{2}}{7\sqrt{3}} \\
 &= \frac{\sqrt{2} \times \sqrt{3}}{7\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{6}}{21}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \quad \left(-\frac{2}{\sqrt{5}}\right) \div \frac{\sqrt{2}}{2} \\
 &= -\frac{2}{\sqrt{5}} \times \frac{2}{\sqrt{2}} = -\frac{2 \times 2}{\sqrt{5} \times \sqrt{2}} \\
 &= -\frac{4}{\sqrt{10}} = -\frac{4 \times \sqrt{10}}{\sqrt{10} \times \sqrt{10}} \\
 &= -\frac{4\sqrt{10}}{10} = -\frac{2\sqrt{10}}{5}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{4} \quad \sqrt{\frac{3}{4}} \div \sqrt{6} \\
 &= \sqrt{\frac{3}{4}} \times \sqrt{\frac{1}{6}} = \sqrt{\frac{3 \times 1}{4 \times 6}} \\
 &= \sqrt{\frac{1}{8}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}} \\
 &= \frac{1 \times \sqrt{2}}{2\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{4}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \quad \frac{\sqrt{2}}{2} - \frac{1}{\sqrt{2}} \\
 &= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{6} \quad \frac{2}{\sqrt{3}} + \frac{\sqrt{3}}{2} \\
 &= \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} + \frac{\sqrt{3}}{2} \\
 &= \frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{2} \\
 &= \frac{7\sqrt{3}}{6}
 \end{aligned}$$

$$\begin{aligned}
\textcircled{7} \quad & \sqrt{\frac{3}{2}} - \frac{1}{\sqrt{6}} \\
&= \frac{\sqrt{3}}{\sqrt{2}} - \frac{\sqrt{6}}{\sqrt{6} \times \sqrt{6}} \\
&= \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} - \frac{\sqrt{6}}{6} \\
&= \frac{\sqrt{6}}{2} - \frac{\sqrt{6}}{6} \\
&= \frac{\sqrt{6}}{3}
\end{aligned}$$

$$\begin{aligned}
\textcircled{8} \quad & \sqrt{\frac{5}{2}} - \sqrt{\frac{2}{5}} \\
&= \frac{\sqrt{5}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{5}} \\
&= \frac{\sqrt{10}}{2} - \frac{\sqrt{10}}{5} \\
&= \frac{3\sqrt{10}}{10}
\end{aligned}$$

$$\begin{aligned}
\textcircled{9} \quad & \frac{1}{\sqrt{6}} + \frac{\sqrt{2}}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{2}} \\
&= \frac{\sqrt{6}}{(\sqrt{6})^2} + \frac{\sqrt{2} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} - \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\
&= \frac{\sqrt{6}}{6} + \frac{\sqrt{6}}{3} - \frac{\sqrt{6}}{2} \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\textcircled{10} \quad & \sqrt{28} - \frac{5}{2\sqrt{7}} - \frac{\sqrt{5}}{\sqrt{35}} \\
&= 2\sqrt{7} - \frac{5\sqrt{7}}{2 \times 7} - \frac{\sqrt{5} \times \sqrt{35}}{35} \\
&= 2\sqrt{7} - \frac{5\sqrt{7}}{14} - \frac{\sqrt{7}}{7} \\
&= \frac{28 - 5 - 2}{14} \sqrt{7} \\
&= \frac{3\sqrt{7}}{2}
\end{aligned}$$

$$\begin{aligned}
\textcircled{11} \quad & \sqrt{24} + \sqrt{\frac{2}{3}} + 2\sqrt{\frac{3}{8}} \\
&= 2\sqrt{6} + \frac{\sqrt{6}}{3} + \frac{2\sqrt{3}}{2\sqrt{2}} \\
&= 2\sqrt{6} + \frac{\sqrt{6}}{3} + \frac{\sqrt{6}}{2} \\
&= \frac{12\sqrt{6}}{6} + \frac{2\sqrt{6}}{6} + \frac{3\sqrt{6}}{6} \\
&= \frac{17\sqrt{6}}{6}
\end{aligned}$$

$$\begin{aligned}
\textcircled{12} \quad & \frac{1}{2\sqrt{15}} - \frac{\sqrt{3}}{3\sqrt{5}} + \frac{3\sqrt{5}}{2\sqrt{3}} \\
&= \frac{\sqrt{15}}{30} - \frac{\sqrt{15}}{15} + \frac{\sqrt{15}}{2} \\
&= \frac{\sqrt{15}}{30} - \frac{2\sqrt{15}}{30} + \frac{15\sqrt{15}}{30} \\
&= \frac{14\sqrt{15}}{30} \\
&= \frac{7\sqrt{15}}{15}
\end{aligned}$$

$$\begin{aligned}
(3) \textcircled{1} \quad & \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\
&= \frac{2\sqrt{2}}{2} \\
&= \sqrt{2} \\
&\div 1.41
\end{aligned}$$

$$\begin{aligned}
\textcircled{2} \quad & \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\
&= \frac{12\sqrt{3}}{3} \\
&= 4\sqrt{3} \\
&\div 4 \times 1.73 \\
&= 6.92
\end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \quad \frac{1}{\sqrt{50}} &= \frac{1}{5\sqrt{2}} \\
 &= \frac{\sqrt{2}}{5\sqrt{2} \times \sqrt{2}} \\
 &= \frac{\sqrt{2}}{10} \\
 &\div \mathbf{0.14}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{4} \quad \frac{2\sqrt{15}}{\sqrt{10}} &= \frac{2\sqrt{15} \times \sqrt{10}}{\sqrt{10} \times \sqrt{10}} \\
 &= \frac{2 \times 5 \times \sqrt{3} \times \sqrt{2}}{10} \\
 &= \sqrt{3} \times \sqrt{2} \\
 &\div 2.4393 \\
 &\div \mathbf{2.44}
 \end{aligned}$$

<注> これは $\sqrt{6} = 2.44949\dots$ となるはずであるが、 $\sqrt{2}$ 、 $\sqrt{3}$ の小数点 3 位以下を近似しているためにこのように誤差が生じている。

$$\begin{aligned}
 \text{【5】 (1)} \quad &\sqrt{2}(\sqrt{3} - \sqrt{5}) \\
 &= \sqrt{2} \times \sqrt{3} - \sqrt{2} \times \sqrt{5} \\
 &= \mathbf{\sqrt{6} - \sqrt{10}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(2)} \quad &\sqrt{2}(\sqrt{3} + 3\sqrt{2}) \\
 &= \sqrt{2} \times \sqrt{3} + \sqrt{2} \times 3\sqrt{2} \\
 &= \mathbf{\sqrt{6} + 6}
 \end{aligned}$$

$$\begin{aligned}
 \text{(3)} \quad &2\sqrt{5}(\sqrt{2} + 2\sqrt{10}) \\
 &= 2\sqrt{5} \times \sqrt{2} + 2\sqrt{5} \times 2\sqrt{10} \\
 &= \mathbf{2\sqrt{10} + 20\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(4)} \quad &(\sqrt{20} - \sqrt{15}) \div \sqrt{5} \\
 &= \sqrt{\frac{20}{5}} - \sqrt{\frac{15}{5}} \\
 &= \mathbf{2 - \sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(5)} \quad &(4\sqrt{50} - 2\sqrt{32}) \div (-2\sqrt{2}) \\
 &= -\frac{4\sqrt{50}}{2\sqrt{2}} + \frac{2\sqrt{32}}{2\sqrt{2}} \\
 &= -2\sqrt{\frac{50}{2}} + \sqrt{\frac{32}{2}} \\
 &= -10 + 4 \\
 &= \mathbf{-6}
 \end{aligned}$$

$$\begin{aligned}
 \text{(6)} \quad &\frac{\sqrt{5} - \sqrt{2}}{\sqrt{2}} \\
 &= \frac{\sqrt{2}(\sqrt{5} - \sqrt{2})}{\sqrt{2} \times \sqrt{2}} \\
 &= \frac{\sqrt{2} \times \sqrt{5} - \sqrt{2} \times \sqrt{2}}{2} \\
 &= \mathbf{\frac{\sqrt{10} - 2}{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{【6】 (1)} \quad &2\sqrt{3} \times \sqrt{6} - \sqrt{8} \\
 &= 6\sqrt{2} - 2\sqrt{2} \\
 &= \mathbf{4\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(2)} \quad &\sqrt{2} + \sqrt{3} \times \sqrt{6} - \sqrt{8} \\
 &= \sqrt{2} + 3\sqrt{2} - 2\sqrt{2} \\
 &= \mathbf{2\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(3)} \quad &\sqrt{72} \div 2\sqrt{3} - \sqrt{54} \\
 &= \frac{6\sqrt{2}}{2\sqrt{3}} - 3\sqrt{6} \\
 &= \sqrt{6} - 3\sqrt{6} \\
 &= \mathbf{-2\sqrt{6}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(4)} \quad &\sqrt{27} - \frac{\sqrt{6}}{\sqrt{2}} + \sqrt{15} \times \sqrt{5} \\
 &= 3\sqrt{3} - \sqrt{3} + 5\sqrt{3} \\
 &= \mathbf{7\sqrt{3}}
 \end{aligned}$$

$$(5) \quad (2\sqrt{3})^2 - \sqrt{2} \times 3\sqrt{3} \\ = 12 - 3\sqrt{6}$$

$$(6) \quad 3\sqrt{3} \times \sqrt{2} - (-\sqrt{2})^2 \times \sqrt{6} \\ = 3\sqrt{6} - 2\sqrt{6} \\ = \sqrt{6}$$

$$\mathbf{【7】} (1) \quad (\sqrt{3} - \sqrt{5}) \times \sqrt{2} + 2\sqrt{10} \\ = \sqrt{6} - \sqrt{10} + 2\sqrt{10} \\ = \sqrt{6} + \sqrt{10}$$

$$(2) \quad \sqrt{24} + \sqrt{2}(\sqrt{6} - \sqrt{3}) \\ = 2\sqrt{6} + \sqrt{12} - \sqrt{6} \\ = \sqrt{6} + 2\sqrt{3}$$

$$(3) \quad 2\sqrt{5}(\sqrt{3} - \sqrt{5}) + 3\sqrt{\frac{5}{3}} \\ = 2\sqrt{15} - 10 + \sqrt{15} \\ = 3\sqrt{15} - 10$$

$$(4) \quad 7\sqrt{6} - (\sqrt{3} - 3\sqrt{2}) \div \sqrt{3} \\ = 7\sqrt{6} - \frac{\sqrt{3}}{\sqrt{3}} + \frac{3\sqrt{2}}{\sqrt{3}} \\ = 7\sqrt{6} - 1 + \frac{3\sqrt{6}}{3} \\ = 7\sqrt{6} - 1 + \sqrt{6} \\ = 8\sqrt{6} - 1$$

$$(5) \quad \sqrt{5}(2 - \sqrt{6}) + \frac{\sqrt{30} + 2\sqrt{5}}{3\sqrt{6}} \\ = 2\sqrt{5} - \sqrt{30} + \frac{\sqrt{30}}{3\sqrt{6}} + \frac{2\sqrt{5}}{3\sqrt{6}} \\ = 2\sqrt{5} - \sqrt{30} + \frac{\sqrt{5}}{3} + \frac{\sqrt{30}}{9} \\ = \frac{6\sqrt{5}}{3} - \frac{9\sqrt{30}}{9} + \frac{\sqrt{5}}{3} + \frac{\sqrt{30}}{9} \\ = \frac{7\sqrt{5}}{3} - \frac{8\sqrt{30}}{9}$$

$$(6) \quad \frac{2\sqrt{5} - 2\sqrt{2}}{\sqrt{10}} - \frac{4 + 3\sqrt{10}}{\sqrt{2}} \\ = \frac{\sqrt{10}(2\sqrt{5} - 2\sqrt{2})}{\sqrt{10} \times \sqrt{10}} - \frac{\sqrt{2}(4 + 3\sqrt{10})}{\sqrt{2} \times \sqrt{2}} \\ = \frac{10\sqrt{2} - 4\sqrt{5}}{10} - \frac{4\sqrt{2} + 6\sqrt{5}}{2} \\ = \sqrt{2} - \frac{2\sqrt{5}}{5} - 2\sqrt{2} - 3\sqrt{5} \\ = -\sqrt{2} - \frac{17\sqrt{5}}{5}$$

$$(7) \quad \sqrt{27}(\sqrt{12} - 2\sqrt{18} + 3\sqrt{8}) + \sqrt{8}(2\sqrt{6} - \sqrt{72} - \sqrt{24}) \\ = 3\sqrt{3}(2\sqrt{3} - 6\sqrt{2} + 6\sqrt{2}) + 2\sqrt{2}(2\sqrt{6} - 6\sqrt{2} - 2\sqrt{6}) \\ = 3\sqrt{3} \times 2\sqrt{3} + 2\sqrt{2} \times (-6\sqrt{2}) \\ = 18 - 24 \\ = -6$$

$$\begin{aligned}
(8) \quad & \sqrt{(-3)^2} \div \left(-\frac{\sqrt{8}}{4}\right)^3 + (-0.5)^2 + (\sqrt{2})^5 + \sqrt{72} \\
& = 3 \times \left(-\frac{4}{\sqrt{2}}\right) + \frac{1}{4} + 4\sqrt{2} + 6\sqrt{2} \\
& = -6\sqrt{2} + \frac{1}{4} + 4\sqrt{2} + 6\sqrt{2} \\
& = 4\sqrt{2} + \frac{1}{4}
\end{aligned}$$

$$\begin{aligned}
\text{【8】 (1)} \quad & (2\sqrt{2} + \sqrt{3} + \sqrt{2} + 3\sqrt{3}) \times 2 = (3\sqrt{2} + 4\sqrt{3}) \times 2 \\
& = 6\sqrt{2} + 8\sqrt{3}
\end{aligned}$$

よって、まわりの長さは $(6\sqrt{2} + 8\sqrt{3})\text{m}$

(2) 台形の下底を $x\text{cm}$ とすると、

$$\begin{aligned}
\frac{1}{2} \times (2\sqrt{2} + x) \times 3\sqrt{6} &= 24\sqrt{3} \\
2\sqrt{2} + x &= 24\sqrt{3} \times \frac{2}{3\sqrt{6}} \\
2\sqrt{2} + x &= 8\sqrt{2} \\
x &= 6\sqrt{2}
\end{aligned}$$

よって、求める下底の長さは $6\sqrt{2}\text{cm}$

$$\begin{aligned}
(3) \quad \text{体積} &= \sqrt{2} \times \sqrt{10} \times (4 - \sqrt{5}) = 2\sqrt{5}(4 - \sqrt{5}) \\
&= 8\sqrt{5} - 10
\end{aligned}$$

よって、体積は $(8\sqrt{5} - 10)\text{cm}^3$

$$\begin{aligned}
\text{表面積} &= \{\sqrt{2} \times \sqrt{10} + \sqrt{10} \times (4 - \sqrt{5}) + \sqrt{2} \times (4 - \sqrt{5})\} \times 2 \\
&= (2\sqrt{5} + 4\sqrt{10} - 5\sqrt{2} + 4\sqrt{2} - \sqrt{10}) \times 2 \\
&= (2\sqrt{5} + 3\sqrt{10} - \sqrt{2}) \times 2 \\
&= 6\sqrt{10} + 4\sqrt{5} - 2\sqrt{2}
\end{aligned}$$

よって、表面積は $(6\sqrt{10} + 4\sqrt{5} - 2\sqrt{2})\text{cm}^2$

$$\begin{array}{ll}
\text{【9】 (1)} \quad (\sqrt{3} - \sqrt{2})^2 & (2) \quad (\sqrt{6} - 2\sqrt{2})^2 \\
= (\sqrt{3})^2 - 2 \times \sqrt{3} \times \sqrt{2} + (\sqrt{2})^2 & = (\sqrt{6})^2 - 2 \times \sqrt{6} \times 2\sqrt{2} + (2\sqrt{2})^2 \\
= 3 - 2\sqrt{6} + 2 & = 6 - 4\sqrt{2 \times 2 \times 3} + 8 \\
= 5 - 2\sqrt{6} & = 14 - 8\sqrt{3}
\end{array}$$

$$\begin{aligned}
 (3) \quad & (\sqrt{5} + 2)(\sqrt{5} - 2) \\
 & = (\sqrt{5})^2 - 2^2 \\
 & = 5 - 4 \\
 & = \mathbf{1}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad & (3\sqrt{2} - \sqrt{3})(3\sqrt{2} + \sqrt{3}) \\
 & = (3\sqrt{2})^2 - (\sqrt{3})^2 \\
 & = 18 - 3 \\
 & = \mathbf{15}
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad & (3\sqrt{7} - \sqrt{3})(3\sqrt{7} - 4\sqrt{3}) \\
 & = (3\sqrt{7})^2 - (4\sqrt{3} + \sqrt{3}) \times 3\sqrt{7} + 4 \times (\sqrt{3})^2 \\
 & = 63 - 5\sqrt{3} \times 3\sqrt{7} + 12 \\
 & = \mathbf{75 - 15\sqrt{21}}
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad & (2\sqrt{10} + 3\sqrt{5})(2\sqrt{10} - 3\sqrt{5}) \\
 & = (2\sqrt{10})^2 - (3\sqrt{5})^2 \\
 & = 40 - 45 \\
 & = \mathbf{-5}
 \end{aligned}$$

【10】 (1) $(\sqrt{3} + \sqrt{2})^2 - 2\sqrt{6}$

$$\begin{aligned}
 & = (\sqrt{3})^2 + 2 \times \sqrt{3} \times \sqrt{2} + (\sqrt{2})^2 - 2\sqrt{6} \\
 & = (\sqrt{3})^2 + 2\sqrt{6} + (\sqrt{2})^2 - 2\sqrt{6} \\
 & = (\sqrt{3})^2 + (\sqrt{2})^2 \\
 & = 3 + 2 \\
 & = \mathbf{5}
 \end{aligned}$$

(2) $2 = a, \sqrt{3} = b$ とおくと,

$$\begin{aligned}
 (2 - \sqrt{3})^2 + (2 + \sqrt{3})^2 & = (a - b)^2 + (a + b)^2 \\
 & = 2(a^2 + b^2) \\
 & = 2\{2^2 + (\sqrt{3})^2\} \\
 & = 2 \times 7 \\
 & = \mathbf{14}
 \end{aligned}$$

$$\begin{aligned}
(3) \quad (1 - \sqrt{3})^2 + \frac{18}{\sqrt{12}} &= (1 - \sqrt{3})^2 + \frac{18}{2\sqrt{3}} \\
&= (1 - \sqrt{3})^2 + \frac{18\sqrt{3}}{2\sqrt{3} \times \sqrt{3}} \\
&= (1 - \sqrt{3})^2 + 3\sqrt{3} \\
&= 1^2 - 2 \times 1 \times \sqrt{3} + (\sqrt{3})^2 + 3\sqrt{3} \\
&= 1 - 2\sqrt{3} + 3 + 3\sqrt{3} \\
&= 4 + \sqrt{3}
\end{aligned}$$

$$\begin{aligned}
(4) \quad &\left(\sqrt{12} - \frac{3}{\sqrt{3}}\right)^2 + 2\sqrt{2} \\
&= \left(2\sqrt{3} - \frac{3\sqrt{3}}{\sqrt{3} \times \sqrt{3}}\right)^2 + 2\sqrt{2} \\
&= (2\sqrt{3} - \sqrt{3})^2 + 2\sqrt{2} \\
&= (\sqrt{3})^2 + 2\sqrt{2} \\
&= \mathbf{3 + 2\sqrt{2}}
\end{aligned}$$

$$\begin{aligned}
(5) \quad &(3\sqrt{24} - \sqrt{96} + \sqrt{12})^2 \\
&= (6\sqrt{6} - 4\sqrt{6} + 2\sqrt{3})^2 \\
&= (2\sqrt{6} + 2\sqrt{3})^2 \\
&= (2\sqrt{6})^2 + 2 \times 2\sqrt{6} \times 2\sqrt{3} + (2\sqrt{3})^2 \\
&= 24 + 8\sqrt{3} \times 3 \times 2 + 12 \\
&= \mathbf{36 + 24\sqrt{2}}
\end{aligned}$$

$$\begin{aligned}
(6) \quad \sqrt{28} = A, \quad \sqrt{27} - 1 = B \quad \text{とおくと,} \\
(\sqrt{28} + \sqrt{27} - 1)(\sqrt{28} - \sqrt{27} + 1) &= \{\sqrt{28} + (\sqrt{27} - 1)\}\{\sqrt{28} - (\sqrt{27} - 1)\} \\
&= (A + B)(A - B) \\
&= A^2 - B^2 \\
&= (\sqrt{28})^2 - (\sqrt{27} - 1)^2 \\
&= 28 - (27 - 2\sqrt{27} + 1) \\
&= 2\sqrt{27} \\
&= \mathbf{6\sqrt{3}}
\end{aligned}$$

$$\begin{aligned}
(7) \quad &(\sqrt{6} + \sqrt{27} + \sqrt{24})(\sqrt{96} - \sqrt{48}) \\
&= (\sqrt{6} + 3\sqrt{3} + 2\sqrt{6})(4\sqrt{6} - 4\sqrt{3}) \\
&= (3\sqrt{6} + 3\sqrt{3})(4\sqrt{6} - 4\sqrt{3}) \\
&= 3(\sqrt{6} + \sqrt{3}) \times 4(\sqrt{6} - \sqrt{3}) \\
&= 3 \times 4(\sqrt{6} + \sqrt{3})(\sqrt{6} - \sqrt{3}) \\
&= 12(6 - 3) \\
&= \mathbf{36}
\end{aligned}$$

$$\begin{aligned}
(8) \quad & (\sqrt{162} + \sqrt{12} - \sqrt{98})(\sqrt{108} - \sqrt{27} - \sqrt{18}) \\
& = (9\sqrt{2} + 2\sqrt{3} - 7\sqrt{2})(6\sqrt{3} - 3\sqrt{3} - 3\sqrt{2}) \\
& = (2\sqrt{2} + 2\sqrt{3})(3\sqrt{3} - 3\sqrt{2}) \\
& = 2(\sqrt{3} + \sqrt{2}) \times 3(\sqrt{3} - \sqrt{2}) \\
& = 6(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) \\
& = 6(3 - 2) \\
& = \mathbf{6}
\end{aligned}$$

$$\begin{aligned}
\text{【11】 (1)} \quad & \frac{(\sqrt{6} - \sqrt{3})^2}{3} - \frac{2 - 2\sqrt{3}}{\sqrt{6}} \times \frac{\sqrt{12} - \sqrt{24}}{\sqrt{3}} \\
& = \frac{6 - 2\sqrt{18} + 3}{3} - \frac{2\sqrt{6} - 2\sqrt{18}}{6} \times (\sqrt{4} - \sqrt{8}) \\
& = \frac{9 - 6\sqrt{2}}{3} - \frac{\sqrt{6} - 3\sqrt{2}}{3} \times (2 - 2\sqrt{2}) \\
& = \frac{9 - 6\sqrt{2}}{3} - \frac{(\sqrt{6} - 3\sqrt{2})(2 - 2\sqrt{2})}{3} \\
& = \frac{9 - 6\sqrt{2}}{3} - \frac{2\sqrt{6} - 2\sqrt{12} - 6\sqrt{2} + 6 \times 2}{3} \\
& = \frac{9 - 6\sqrt{2}}{3} - \frac{2\sqrt{6} - 4\sqrt{3} - 6\sqrt{2} + 12}{3} \\
& = \frac{9 - 6\sqrt{2} - 2\sqrt{6} + 4\sqrt{3} + 6\sqrt{2} - 12}{3} = \frac{-3 + 4\sqrt{3} - 2\sqrt{6}}{3}
\end{aligned}$$

$$\begin{aligned}
(2) \quad & \sqrt{2} + 3 = A, \quad \sqrt{2} - 3 = B, \quad \sqrt{2} + 2 = C, \quad \sqrt{2} - 2 = D \text{ とおくと,} \\
& \frac{(\sqrt{2} + 2)^2 - (\sqrt{2} - 2)^2}{(\sqrt{2} + 3)^2 - (\sqrt{2} - 3)^2} = \frac{C^2 - D^2}{A^2 - B^2} = \frac{(C + D)(C - D)}{(A + B)(A - B)} \\
& = \frac{(\sqrt{2} + 2 + \sqrt{2} - 2)(\sqrt{2} + 2 - \sqrt{2} + 2)}{(\sqrt{2} + 3 + \sqrt{2} - 3)(\sqrt{2} + 3 - \sqrt{2} + 3)} \\
& = \frac{2\sqrt{2} \times 4}{2\sqrt{2} \times 6} = \frac{\mathbf{2}}{\mathbf{3}}
\end{aligned}$$

$$\begin{aligned}
(3) \quad & (\sqrt{2} - 1)(\sqrt{2} - 2)(\sqrt{2} - 3)(\sqrt{2} - 4) \\
& = (\sqrt{2} - 1)(\sqrt{2} - 4)(\sqrt{2} - 2)(\sqrt{2} - 3) \\
& = \{2 - (4 + 1) \times \sqrt{2} + 4\} \{2 - (3 + 2) \times \sqrt{2} + 6\} \\
& = (6 - 5\sqrt{2})(8 - 5\sqrt{2}) \\
& = 48 - (6 + 8) \times 5\sqrt{2} + 50 \\
& = \mathbf{98 - 70\sqrt{2}}
\end{aligned}$$

(4) $\frac{2}{\sqrt{3}} = A, \frac{3}{\sqrt{6}} = B$ とおくと,

$$\begin{aligned} \left(\frac{2}{\sqrt{3}} + \frac{3}{\sqrt{6}}\right)^2 - \left(\frac{2}{\sqrt{3}} - \frac{3}{\sqrt{6}}\right)^2 &= (A+B)^2 - (A-B)^2 = 4AB \\ &= 4 \times \frac{2}{\sqrt{3}} \times \frac{3}{\sqrt{6}} = \frac{24}{\sqrt{18}} = \frac{24}{3\sqrt{2}} \\ &= \frac{8}{\sqrt{2}} = \frac{8 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{8\sqrt{2}}{2} \\ &= 4\sqrt{2} \end{aligned}$$

【12】 (1) $\sqrt{3}x = \sqrt{24} + \sqrt{27}$

$$x = \frac{1}{\sqrt{3}}(\sqrt{24} + \sqrt{27})$$

$$x = 2\sqrt{2} + 3$$

(2) $\sqrt{5}(x + \sqrt{2}) = 1$

$$\sqrt{5}x + \sqrt{10} = 1$$

$$\sqrt{5}x = 1 - \sqrt{10}$$

$$x = \frac{1}{\sqrt{5}}(1 - \sqrt{10})$$

$$x = \frac{\sqrt{5}}{5} - \sqrt{2}$$

【13】 (1) $(\sqrt{3} + \sqrt{7})^2 = 3 + 2\sqrt{21} + 7 = 10 + 2\sqrt{21}$

$$(2 + \sqrt{6})^2 = 4 + 4\sqrt{6} + 6 = 10 + 4\sqrt{6} = 10 + 2\sqrt{24}$$

$\sqrt{21} < \sqrt{24}$ より,

$$(\sqrt{3} + \sqrt{7})^2 < (2 + \sqrt{6})^2$$

$$\therefore \sqrt{3} + \sqrt{7} < 2 + \sqrt{6}$$

(2) $(\sqrt{13} + \sqrt{19})^2 = 13 + 2\sqrt{13 \times 19} + 19 = 32 + 2\sqrt{13 \times 19}$

これと $8^2 = 64$ との大きさを比較.

$$8^2 - (\sqrt{13} + \sqrt{19})^2 = 32 - 2\sqrt{13 \times 19} = 2(16 - \sqrt{13 \times 19})$$

ここで, $16^2 = 256, 13 \times 19 = 247$ より, $16 > \sqrt{13 \times 19}$

よって,

$$8^2 > (\sqrt{13} + \sqrt{19})^2$$

$$\therefore 8 > \sqrt{13} + \sqrt{19}$$

<別解>

$$8^2 - (\sqrt{13} + \sqrt{19})^2 = 64 - (32 + 2\sqrt{13 \times 19})$$

$$= 32 - 2\sqrt{13 \times 19}$$

$$= (\sqrt{19} - \sqrt{13})^2 > 0$$

以上より, $8 > \sqrt{13} + \sqrt{19}$

【14】根号の中身どうしを比べればよいので、 $15 - \sqrt{6}$ と $5 + 2\sqrt{15}$ との大小を比較すればよい。そのためには、2つの数を引き算して、0 との大小を調べればよい。

$15 - \sqrt{6}$ から $5 + 2\sqrt{15}$ を引いて

$$(15 - \sqrt{6}) - (5 + 2\sqrt{15}) = 10 - (\sqrt{6} + 2\sqrt{15})$$

この式が0より大きいか、小さいかを調べるには、10 と $(\sqrt{6} + 2\sqrt{15})$ との大小を比較すればよい。それぞれを2乗してみると、

$$10^2 = 100$$

$$(\sqrt{6} + 2\sqrt{15})^2 = 6 + 2\sqrt{6} \cdot 2\sqrt{15} + 4 \cdot 15 = 66 + 12\sqrt{10}$$

ここでさらに100 と $66 + 12\sqrt{10}$ との大小を比較するために引いてみると、

$$100 - (66 + 12\sqrt{10}) = 34 - 12\sqrt{10}$$

これが0より大きいか、小さいかを調べるには、34 の2乗と $12\sqrt{10}$ の2乗の大小を比較すればよい。

$$34^2 = 1156$$

$$(12\sqrt{10})^2 = 1440$$

よって、 $12\sqrt{10}$ の方が大きい。

$$100 - (66 + 12\sqrt{10}) = 34 - 12\sqrt{10} < 0$$

$$100 < 66 + 12\sqrt{10}$$

$$10 < \sqrt{6} + 2\sqrt{15}$$

$$(15 - \sqrt{6}) - (5 + 2\sqrt{15}) = 10 - (\sqrt{6} + 2\sqrt{15}) < 0$$

$$15 - \sqrt{6} < 5 + 2\sqrt{15}$$

$$\sqrt{15 - \sqrt{6}} < \sqrt{5 + 2\sqrt{15}}$$

添削課題

$$\text{【1】 (1) } \frac{2}{\sqrt{5}} = \frac{2 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$(2) \frac{3\sqrt{2}}{\sqrt{3}} = \frac{3\sqrt{2} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{3\sqrt{6}}{3} = \sqrt{6}$$

$$(3) \sqrt{\frac{3}{7}} = \frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{3} \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{\sqrt{21}}{7}$$

$$(4) \frac{\sqrt{20}}{\sqrt{27}} = \frac{2\sqrt{5}}{3\sqrt{3}} \\ = \frac{2\sqrt{5} \times \sqrt{3}}{3\sqrt{3} \times \sqrt{3}} = \frac{2\sqrt{15}}{9}$$

$$\text{【2】 (1) } 2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$$

$$(2) -3\sqrt{3} - 6\sqrt{3} = -9\sqrt{3}$$

$$(3) \sqrt{6} - \sqrt{6} - \sqrt{6} = -\sqrt{6}$$

$$(4) 3\sqrt{3} - 8\sqrt{3} + 6\sqrt{3} = \sqrt{3}$$

$$(5) \quad 3\sqrt{2} - 2\sqrt{3} - 4\sqrt{2} + \sqrt{3} \\ = (3-4)\sqrt{2} + (-2+1)\sqrt{3} \\ = -\sqrt{2} - \sqrt{3}$$

$$(6) \quad 3 + 5\sqrt{5} - 3\sqrt{2} - 4\sqrt{5} - 5 + \sqrt{2} \\ = -2 - 2\sqrt{2} + \sqrt{5}$$

$$\text{【3】 (1) } \sqrt{24} - \sqrt{54} = 2\sqrt{6} - 3\sqrt{6} \\ = -\sqrt{6}$$

$$(2) \sqrt{50} + \sqrt{32} = 5\sqrt{2} + 4\sqrt{2} \\ = 9\sqrt{2}$$

$$(3) \quad \sqrt{27} - \sqrt{108} + \sqrt{75} \\ = 3\sqrt{3} - 6\sqrt{3} + 5\sqrt{3} \\ = 2\sqrt{3}$$

$$(4) \quad \sqrt{48} - \sqrt{72} - \sqrt{147} + \sqrt{128} \\ = 4\sqrt{3} - 6\sqrt{2} - 7\sqrt{3} + 8\sqrt{2} \\ = -3\sqrt{3} + 2\sqrt{2}$$

$$(5) \quad \sqrt{\frac{1}{2}} + 3\sqrt{2} \\ = \frac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} + 3\sqrt{2} \\ = \frac{\sqrt{2}}{2} + \frac{6\sqrt{2}}{2} \\ = \frac{7\sqrt{2}}{2}$$

$$(6) \quad -\sqrt{\frac{2}{3}} + \sqrt{27} - \frac{\sqrt{12}}{3} \\ = -\frac{\sqrt{2} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} + 3\sqrt{3} - \frac{2\sqrt{3}}{3} \\ = \frac{7\sqrt{3} - \sqrt{6}}{3}$$

$$\begin{aligned}
(7) \quad & \frac{1}{\sqrt{3}} - \frac{4}{\sqrt{12}} + \frac{6}{\sqrt{8}} \\
&= \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} - \frac{4\sqrt{3}}{2\sqrt{3} \times \sqrt{3}} + \frac{6\sqrt{2}}{2\sqrt{2} \times \sqrt{2}} \\
&= \frac{\sqrt{3}}{3} - \frac{2\sqrt{3}}{3} + \frac{3\sqrt{2}}{2} \\
&= -\frac{\sqrt{3}}{3} + \frac{3\sqrt{2}}{2}
\end{aligned}$$

$$\begin{aligned}
(8) \quad & \sqrt{\frac{5}{2}} - \sqrt{\frac{18}{5}} + \frac{3}{\sqrt{10}} \\
&= \frac{\sqrt{5}}{\sqrt{2}} - \frac{\sqrt{18}}{\sqrt{5}} + \frac{3}{\sqrt{10}} \\
&= \frac{\sqrt{5} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} - \frac{3\sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} + \frac{3\sqrt{10}}{\sqrt{10} \times \sqrt{10}} \\
&= \frac{\sqrt{10}}{2} - \frac{3\sqrt{10}}{5} + \frac{3\sqrt{10}}{10} \\
&= \frac{5\sqrt{10} - 6\sqrt{10} + 3\sqrt{10}}{10} \\
&= \frac{2\sqrt{10}}{10} \\
&= \frac{\sqrt{10}}{5}
\end{aligned}$$

[4] (1) $\sqrt{2}(\sqrt{10} - \sqrt{3})$

$$\begin{aligned}
&= \sqrt{2} \times \sqrt{10} - \sqrt{2} \times \sqrt{3} \\
&= \sqrt{20} - \sqrt{6} \\
&= 2\sqrt{5} - \sqrt{6}
\end{aligned}$$

(2) $(\sqrt{21} + \sqrt{15}) \div (-\sqrt{3})$

$$\begin{aligned}
&= -\sqrt{\frac{21}{3}} - \sqrt{\frac{15}{3}} \\
&= -\sqrt{7} - \sqrt{5}
\end{aligned}$$

(3) $3\sqrt{2} \times \sqrt{6} - 5\sqrt{3} = 3\sqrt{12} - 5\sqrt{3}$

$$\begin{aligned}
&= 6\sqrt{3} - 5\sqrt{3} \\
&= \sqrt{3}
\end{aligned}$$

$$\begin{aligned}
(4) \quad & \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}} \\
&= \frac{(\sqrt{3}-\sqrt{2}) \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{(\sqrt{3}+\sqrt{2}) \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\
&= \frac{\sqrt{6}-2}{2} + \frac{3+\sqrt{6}}{3} \\
&= \frac{3\sqrt{6}-6+6+2\sqrt{6}}{6} \\
&= \frac{5\sqrt{6}}{6}
\end{aligned}$$

$$\begin{aligned}
\text{【5】 (1)} \quad & (\sqrt{6}-\sqrt{2})^2 & (2) \quad & (3\sqrt{5}-2\sqrt{3})(3\sqrt{5}+2\sqrt{3}) \\
& = (\sqrt{6})^2 - 2 \times \sqrt{6} \times \sqrt{2} + (\sqrt{2})^2 & & = (3\sqrt{5})^2 - (2\sqrt{3})^2 \\
& = 6 - 4\sqrt{3} + 2 & & = 45 - 12 \\
& = \mathbf{8 - 4\sqrt{3}} & & = \mathbf{33}
\end{aligned}$$

$$\begin{aligned}
(3) \quad & (2\sqrt{3}-\sqrt{2})(2\sqrt{3}+3\sqrt{2}) \\
& = (2\sqrt{3})^2 + (-\sqrt{2}+3\sqrt{2}) \times 2\sqrt{3} - \sqrt{2} \times 3\sqrt{2} \\
& = 12 + 4\sqrt{6} - 6 \\
& = \mathbf{6 + 4\sqrt{6}}
\end{aligned}$$

$$\begin{aligned}
(4) \quad & (2+\sqrt{2})^2 - (2-\sqrt{2})^2 \\
& = (4+4\sqrt{2}+2) - (4-4\sqrt{2}+2) \\
& = 6+4\sqrt{2}-6+4\sqrt{2} \\
& = \mathbf{8\sqrt{2}}
\end{aligned}$$

<別解>

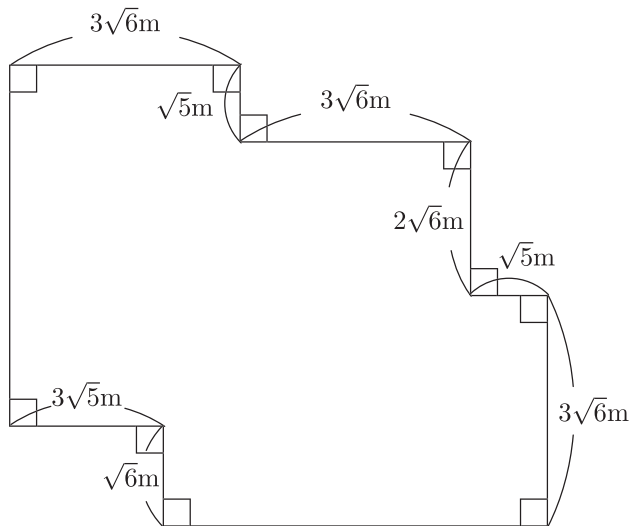
$$\begin{aligned}
2+2\sqrt{2} &= A, \quad 2-\sqrt{2} = B \quad \text{とおくと} \\
(\text{与式}) &= A^2 - B^2 \\
&= (A+B)(A-B) \\
&= \{(2+\sqrt{2})+(2-\sqrt{2})\}\{(2+\sqrt{2})-(2-\sqrt{2})\} \\
&= 4 \times 2\sqrt{2} \\
&= \mathbf{8\sqrt{2}}
\end{aligned}$$

$$\begin{aligned}
(5) \quad & (1 + \sqrt{2} - \sqrt{3})(1 - \sqrt{2} + \sqrt{3}) \\
& = \{1 + (\sqrt{2} - \sqrt{3})\} \{1 - (\sqrt{2} - \sqrt{3})\} \\
& = 1^2 - (\sqrt{2} - \sqrt{3})^2 \\
& = 1 - (5 - 2\sqrt{6}) \\
& = -4 + 2\sqrt{6}
\end{aligned}$$

$$\begin{aligned}
(6) \quad & (\sqrt{8} + \sqrt{24} - 3\sqrt{2}) \left(\frac{\sqrt{150}}{2} + \frac{2}{\sqrt{2}} - \sqrt{\frac{3}{2}} \right) \\
& = (2\sqrt{2} + 2\sqrt{6} - 3\sqrt{2}) \left(\frac{5\sqrt{6}}{2} + \frac{2\sqrt{2}}{2} - \frac{\sqrt{3}}{\sqrt{2}} \right) \\
& = (2\sqrt{6} - \sqrt{2}) \left(\frac{5}{2}\sqrt{6} + \sqrt{2} - \frac{\sqrt{6}}{2} \right) \\
& = (2\sqrt{6} - \sqrt{2})(2\sqrt{6} + \sqrt{2}) \\
& = 24 - 2 \\
& = \mathbf{22}
\end{aligned}$$

[6] (周囲の長さ) $= 2(3\sqrt{6} + 3\sqrt{6} + \sqrt{5}) + 2(\sqrt{5} + 2\sqrt{6} + 3\sqrt{6})$
 $= 2(6\sqrt{6} + \sqrt{5}) + 2(\sqrt{5} + 5\sqrt{6})$
 $= \mathbf{22\sqrt{6} + 4\sqrt{5}(m)}$

(面積) $= (6\sqrt{6} + \sqrt{5})(5\sqrt{6} + \sqrt{5}) - \sqrt{5} \times 3\sqrt{6} - (\sqrt{5})^2 - 2\sqrt{6} \times \sqrt{5} - 3\sqrt{5} \times \sqrt{6}$
 $= 30(\sqrt{6})^2 + 11\sqrt{30} + (\sqrt{5})^2 - 8\sqrt{30} - (\sqrt{5})^2$
 $= \mathbf{180 + 3\sqrt{30}(m^2)}$



小テスト

- 【1】** (1) $\sqrt{6}$
(2) $-2\sqrt{10}$
(3) $6\sqrt{42}$
(4) $-2\sqrt{3}$
(5) $3\sqrt{10}$
(6) $4\sqrt{3}$
(7) $\sqrt{5}$
(8) 2
(9) $2\sqrt{2}$
(10) $\sqrt{3}$

16章 平方根 (3)

問題

$$\begin{aligned} \text{【1】 (1)} \quad & 2\sqrt{3} \times \sqrt{2} - 5\sqrt{6} \\ & = 2\sqrt{6} - 5\sqrt{6} \\ & = -3\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{(2)} \quad & \sqrt{8} - \sqrt{3} \times \sqrt{24} \\ & = 2\sqrt{2} - \sqrt{3 \times 3 \times 2^3} \\ & = 2\sqrt{2} - 6\sqrt{2} \\ & = -4\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{(3)} \quad & \sqrt{3}(2\sqrt{6} - \sqrt{2}) - \sqrt{32} \\ & = 6\sqrt{2} - \sqrt{6} - 4\sqrt{2} \\ & = 2\sqrt{2} - \sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{(4)} \quad & \frac{2\sqrt{5} - \sqrt{12}}{2} + \sqrt{45} + 2\sqrt{27} \\ & = \frac{2\sqrt{5} - 2\sqrt{3}}{2} + 3\sqrt{5} + 6\sqrt{3} \\ & = \sqrt{5} - \sqrt{3} + 3\sqrt{5} + 6\sqrt{3} \\ & = 4\sqrt{5} + 5\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{(5)} \quad & (2 + \sqrt{3})^2 \\ & = 2^2 + 2 \times 2 \times \sqrt{3} + (\sqrt{3})^2 \\ & = 4 + 4\sqrt{3} + 3 \\ & = 7 + 4\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{(6)} \quad & (2\sqrt{2} - \sqrt{5})^2 \\ & = (2\sqrt{2})^2 - 2 \times 2\sqrt{2} \times \sqrt{5} + (\sqrt{5})^2 \\ & = 8 - 4\sqrt{10} + 5 \\ & = 13 - 4\sqrt{10} \end{aligned}$$

$$\begin{aligned} \text{(7)} \quad & (3\sqrt{3} - \sqrt{2})(3\sqrt{3} + \sqrt{2}) \\ & = (3\sqrt{3})^2 - (\sqrt{2})^2 \\ & = 27 - 2 \\ & = 25 \end{aligned}$$

$$\begin{aligned} \text{(8)} \quad & (\sqrt{6} - 2)(\sqrt{6} + 3) \\ & = (\sqrt{6})^2 + \sqrt{6} - 6 \\ & = \sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{【2】 (1)} \quad & \sqrt{18} - 2\sqrt{8} = 3\sqrt{2} - 4\sqrt{2} \\ & = -\sqrt{2} \\ & = -1.414 \end{aligned}$$

$$\begin{aligned} \text{(2)} \quad & \frac{(\sqrt{2})^3 - 2}{\sqrt{2}} = \frac{2\sqrt{2} - 2}{\sqrt{2}} \\ & = 2 - \sqrt{2} \\ & = 2 - 1.414 \\ & = 0.586 \end{aligned}$$

$$\begin{aligned} \text{(3)} \quad & (\sqrt{8} + \sqrt{12})(-\sqrt{18} + \sqrt{27}) = (2\sqrt{2} + 2\sqrt{3})(-3\sqrt{2} + 3\sqrt{3}) \\ & = 2(\sqrt{3} + \sqrt{2}) \times 3(\sqrt{3} - \sqrt{2}) \\ & = 6(3 - 2) \\ & = 6 \end{aligned}$$

$$\begin{aligned}
(4) \quad & -\sqrt{12} + (\sqrt{3})^3 + \sqrt{6} \times \sqrt{3} + (-\sqrt{2})^3 = -2\sqrt{3} + 3\sqrt{3} + 3\sqrt{2} - 2\sqrt{2} \\
& = \sqrt{3} + \sqrt{2} \\
& = 1.732 + 1.414 \\
& = \mathbf{3.146}
\end{aligned}$$

【3】 (1) $3 - \sqrt{2} = A$ とおくと,

$$\begin{aligned}
(3 - \sqrt{2})^2 - 6(3 - \sqrt{2}) + 9 &= A^2 - 6A + 9 = (A - 3)^2 \\
&= (3 - \sqrt{2} - 3)^2 \\
&= \mathbf{2}
\end{aligned}$$

(2) $2 - 3\sqrt{2} = A$ とおくと,

$$\begin{aligned}
(2 - 3\sqrt{2})^2 - 4(2 - 3\sqrt{2}) + 4 &= A^2 - 4A + 4 = (A - 2)^2 \\
&= (2 - 3\sqrt{2} - 2)^2 \\
&= \mathbf{18}
\end{aligned}$$

(3) $\sqrt{10} + 2 = A$, $2 - \sqrt{10} = B$ とおくと,

$$\begin{aligned}
(\sqrt{10} + 2)^2 - (2 - \sqrt{10})^2 &= A^2 - B^2 = (A + B)(A - B) \\
&= \{(\sqrt{10} + 2) + (2 - \sqrt{10})\}\{(\sqrt{10} + 2) - (2 - \sqrt{10})\} \\
&= 4 \times 2\sqrt{10} \\
&= \mathbf{8\sqrt{10}}
\end{aligned}$$

(4) $\sqrt{15} - \sqrt{3} = A$, $\sqrt{15} + \sqrt{3} = B$ とおくと,

$$\begin{aligned}
(\sqrt{15} - \sqrt{3})^2 - (\sqrt{15} + \sqrt{3})^2 &= A^2 - B^2 \\
&= (A + B)(A - B) \\
&= \{(\sqrt{15} - \sqrt{3}) + (\sqrt{15} + \sqrt{3})\}\{(\sqrt{15} - \sqrt{3}) - (\sqrt{15} + \sqrt{3})\} \\
&= 2\sqrt{15} \times (-2\sqrt{3}) \\
&= -4\sqrt{3 \times 3 \times 5} \\
&= \mathbf{-12\sqrt{5}}
\end{aligned}$$

(5) $\sqrt{7} - 5 = A$ とおくと,

$$\begin{aligned}
(\sqrt{7} - 5)^2 + 8(\sqrt{7} - 5) + 15 &= A^2 + 8A + 15 = (A + 5)(A + 3) \\
&= (\sqrt{7} - 5 + 5)(\sqrt{7} - 5 + 3) \\
&= \sqrt{7}(\sqrt{7} - 2) \\
&= \mathbf{7 - 2\sqrt{7}}
\end{aligned}$$

$$\begin{aligned}
(6) \quad 2\sqrt{5} - 1 = A \text{ とおくと,} \\
(2\sqrt{5} - 1)^2 + 3(2\sqrt{5} - 1) + 2 &= A^2 + 3A + 2 = (A + 1)(A + 2) \\
&= (2\sqrt{5} - 1 + 1)(2\sqrt{5} - 1 + 2) \\
&= 2\sqrt{5}(2\sqrt{5} + 1) \\
&= \mathbf{20 + 2\sqrt{5}}
\end{aligned}$$

$$\begin{aligned}
\mathbf{【4】} \quad (1) \quad \frac{1}{\sqrt{3} + \sqrt{2}} &= \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} \\
&= \frac{\sqrt{3} - \sqrt{2}}{3 - 2} \\
&= \mathbf{\sqrt{3} - \sqrt{2}}
\end{aligned}$$

$$\begin{aligned}
(2) \quad \frac{\sqrt{2}}{2\sqrt{2} + 3} &= \frac{\sqrt{2}(2\sqrt{2} - 3)}{(2\sqrt{2} + 3)(2\sqrt{2} - 3)} \\
&= \frac{4 - 3\sqrt{2}}{(2\sqrt{2})^2 - 3^2} \\
&= \frac{4 - 3\sqrt{2}}{8 - 9} \\
&= \frac{4 - 3\sqrt{2}}{-1} \\
&= \mathbf{3\sqrt{2} - 4}
\end{aligned}$$

$$\begin{aligned}
(3) \quad \frac{22}{2\sqrt{3} - 1} \\
&= \frac{22(2\sqrt{3} + 1)}{(2\sqrt{3} - 1)(2\sqrt{3} + 1)} \\
&= \frac{44\sqrt{3} + 22}{(2\sqrt{3})^2 - 1^2} \\
&= \frac{44\sqrt{3} + 22}{12 - 1} \\
&= \frac{44\sqrt{3} + 22}{11} \\
&= \mathbf{4\sqrt{3} + 2}
\end{aligned}$$

$$\begin{aligned}
(4) \quad \frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}} \\
&= \frac{(3\sqrt{2} + 2\sqrt{3})^2}{(3\sqrt{2} - 2\sqrt{3})(3\sqrt{2} + 2\sqrt{3})} \\
&= \frac{(3\sqrt{2})^2 + 2 \times 3\sqrt{2} \times 2\sqrt{3} + (2\sqrt{3})^2}{(3\sqrt{2})^2 - (2\sqrt{3})^2} \\
&= \frac{18 + 12\sqrt{6} + 12}{18 - 12} \\
&= \frac{30 + 12\sqrt{6}}{6} \\
&= \mathbf{5 + 2\sqrt{6}}
\end{aligned}$$

$$\begin{aligned}
(5) \quad & \frac{2}{2+\sqrt{2}} - \frac{2}{\sqrt{2}-1} \\
&= \frac{2}{2+\sqrt{2}} - \frac{2\sqrt{2}}{(\sqrt{2}-1)\times\sqrt{2}} = \frac{2}{2+\sqrt{2}} - \frac{2\sqrt{2}}{2-\sqrt{2}} \\
&= \frac{2(2-\sqrt{2})-2\sqrt{2}(2+\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})} = \frac{2(2-\sqrt{2})-2(2\sqrt{2}+2)}{4-2} \\
&= (2-\sqrt{2}) - (2\sqrt{2}+2) = 2-\sqrt{2}-2\sqrt{2}-2 = -3\sqrt{2}
\end{aligned}$$

$$\begin{aligned}
(6) \quad & \frac{5}{1+\sqrt{5}} - \frac{3}{1-\sqrt{5}} \\
&= \frac{5(1-\sqrt{5})-3(1+\sqrt{5})}{(1+\sqrt{5})(1-\sqrt{5})} = \frac{5-5\sqrt{5}-3-3\sqrt{5}}{1-5} \\
&= \frac{2-8\sqrt{5}}{-4} = \frac{4\sqrt{5}-1}{2}
\end{aligned}$$

【5】 (1) $\sqrt{4} < \sqrt{6} < \sqrt{9}$ より, $2 < \sqrt{6} < 3$

よって, $a = 2$, $b = \sqrt{6} - 2$

$\therefore 2a + b = 4 + \sqrt{6} - 2 = \sqrt{6} + 2$

(2) $\sqrt{9} < \sqrt{10} < \sqrt{16}$ より, $3 < \sqrt{10} < 4$

よって, $a = \sqrt{10} - 3$

$a^2 + 6a = a(a+6)$

$= (\sqrt{10} - 3)(\sqrt{10} + 3)$

$= 10 - 9$

$= 1$

(3) $3\sqrt{5} = \sqrt{45}$

$\sqrt{36} < \sqrt{45} < \sqrt{49}$ より, $6 < \sqrt{45} < 7$

よって, $a = 6$, $b = 3\sqrt{5} - 6$

$2ab + b^2 = b(2a + b)$

$= (3\sqrt{5} - 6)(3\sqrt{5} + 6)$

$= 45 - 36$

$= 9$

(4) $5\sqrt{2} = \sqrt{50}$, $7 = \sqrt{49} < \sqrt{50} < \sqrt{64} = 8$ より, $a = 7$

よって, $b = 5\sqrt{2} - 7$

$\frac{a}{b^2 + 14b} = \frac{a}{b(b+14)}$

$= \frac{7}{(5\sqrt{2} - 7)(5\sqrt{2} + 7)}$

$= \frac{7}{50 - 49}$

$= 7$

$$\begin{aligned}
(5) \quad & (\sqrt{6} + \sqrt{2})^2 = 8 + 2\sqrt{12} = 8 + \sqrt{48} \\
& \sqrt{36} < \sqrt{48} < \sqrt{49} \text{ より, } 6 < \sqrt{48} < 7 \\
& \therefore 8 + 6 < 8 + \sqrt{48} < 8 + 7 \\
& 14 < (\sqrt{6} + \sqrt{2})^2 < 15 \\
& \therefore 9 < (\sqrt{6} + \sqrt{2})^2 < 16 \\
& 3 < \sqrt{6} + \sqrt{2} < 4 \\
& \text{よつて, } a = 3, b = \sqrt{6} + \sqrt{2} - 3 \\
& \frac{4}{a+b} - a + b = \frac{4}{\sqrt{6} + \sqrt{2}} - 3 + (\sqrt{6} + \sqrt{2} - 3) \\
& = \frac{4(\sqrt{6} - \sqrt{2})}{4} - 3 + \sqrt{6} + \sqrt{2} - 3 \\
& = \sqrt{6} - \sqrt{2} + \sqrt{6} + \sqrt{2} - 6 \\
& = \mathbf{2\sqrt{6} - 6}
\end{aligned}$$

$$\text{【6】 (1) } x = \sqrt{3} - 1 \text{ より, } x + 1 = \sqrt{3} \quad \therefore (x + 1)^2 = 3 \\
\therefore x^2 + 2x + 1 = \mathbf{3}$$

$$\begin{aligned}
(2) \quad & x = \sqrt{3} + 2 \text{ より, } x - 2 = \sqrt{3} \quad \therefore (x - 2)^2 = 3 \\
& x^2 - 4x + 4 = 3 \\
& x^2 - 4x + 1 = \mathbf{0}
\end{aligned}$$

$$\begin{aligned}
(3) \quad & x = \sqrt{3} + 1, y = \sqrt{3} - 1 \text{ より,} \\
& x + y = (\sqrt{3} + 1) + (\sqrt{3} - 1) = 2\sqrt{3} \\
& xy = (\sqrt{3} + 1)(\sqrt{3} - 1) = 3 - 1 = 2 \\
& \text{ゆえに, } x^2 + y^2 = (x + y)^2 - 2xy = (2\sqrt{3})^2 - 2 \times 2 = 12 - 4 = \mathbf{8}
\end{aligned}$$

$$\begin{aligned}
(4) \quad & x = \sqrt{3} + \sqrt{5}, y = \sqrt{3} - \sqrt{5} \text{ より,} \\
& x + y = (\sqrt{3} + \sqrt{5}) + (\sqrt{3} - \sqrt{5}) = 2\sqrt{3} \\
& x - y = (\sqrt{3} + \sqrt{5}) - (\sqrt{3} - \sqrt{5}) = 2\sqrt{5} \\
& \text{ゆえに, } x^2 - y^2 = (x + y)(x - y) = 2\sqrt{3} \times 2\sqrt{5} = \mathbf{4\sqrt{15}}
\end{aligned}$$

$$\begin{aligned}
\text{【7】 (1) } \quad & x(2x - 5y) + y(x + 2y) = 2x^2 - 5xy + xy + 2y^2 \\
& = 2x^2 - 4xy + 2y^2 \\
& = 2(x^2 - 2xy + y^2) \\
& = 2(x - y)^2
\end{aligned}$$

$$\begin{aligned}
& \text{ここで, } x - y = (\sqrt{5} + \sqrt{3}) - (\sqrt{5} - \sqrt{3}) = 2\sqrt{3} \text{ より,} \\
& (\text{求値式}) = 2(x - y)^2 = 2 \times (2\sqrt{3})^2 = \mathbf{24}
\end{aligned}$$

$$\begin{aligned}
(2) \quad & x + y = (2 + \sqrt{2}) + (\sqrt{2} - 1) = 2\sqrt{2} + 1 \\
& xy = (2 + \sqrt{2})(\sqrt{2} - 1) = \sqrt{2}(\sqrt{2} + 1)(\sqrt{2} - 1) = \sqrt{2} \times 1 = \sqrt{2} \text{ より,} \\
& \frac{1}{4}x^2 + xy + \frac{1}{4}y^2 = \frac{1}{4}(x^2 + 4xy + y^2) = \frac{1}{4}\{(x + y)^2 + 2xy\} \\
& = \frac{1}{4}\{(2\sqrt{2} + 1)^2 + 2\sqrt{2}\} = \frac{1}{4}(8 + 4\sqrt{2} + 1 + 2\sqrt{2}) \\
& = \frac{\mathbf{9 + 6\sqrt{2}}}{4}
\end{aligned}$$

$$(3) \quad x + y = \frac{\sqrt{2} + 1}{\sqrt{3}} + \frac{\sqrt{2} - 1}{\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{3}}$$

$$xy = \frac{\sqrt{2} + 1}{\sqrt{3}} \times \frac{\sqrt{2} - 1}{\sqrt{3}} = \frac{2 - 1}{3} = \frac{1}{3} \text{ より,}$$

$$x^2 + xy + y^2 = (x + y)^2 - xy = \left(\frac{2\sqrt{2}}{\sqrt{3}}\right)^2 - \frac{1}{3} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

$$(4) \quad \left(x + \frac{1}{x}\right)^2 = x^2 + 2 \times x \times \frac{1}{x} + \frac{1}{x^2} = x^2 + 2 + \frac{1}{x^2} \text{ より,}$$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = (\sqrt{5} - 2)^2 - 2 = 5 - 4\sqrt{5} + 4 - 2 = 7 - 4\sqrt{5}$$

$$(5) \quad \left(x - \frac{1}{x}\right)^2 = x^2 - 2 \times x \times \frac{1}{x} + \frac{1}{x^2} = x^2 - 2 + \frac{1}{x^2} \text{ より,}$$

$$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2 = (\sqrt{2} + \sqrt{5})^2 + 2$$

$$= 2 + 2\sqrt{10} + 5 + 2 = 9 + 2\sqrt{10}$$

$$\mathbf{【8】} (1) \quad y = \sqrt{a^2 - 4a + 4} + \sqrt{a^2 + 4a + 4}$$

$$= \sqrt{(a - 2)^2} + \sqrt{(a + 2)^2}$$

$$= |a - 2| + |a + 2|$$

よって,

$$y = \begin{cases} (a - 2) + (a + 2) = 2a & (2 \leq a) \\ -(a - 2) + (a + 2) = 4 & (-2 \leq a < 2) \\ -(a - 2) - (a + 2) = -2a & (a < -2) \end{cases}$$

(2) (1) より

$$a = 2 + \sqrt{2} \text{ のとき, } y = 2a = 4 + 2\sqrt{2}$$

$$a = 2 - \sqrt{2} \text{ のとき, } y = 4$$

$$a = -2 - \sqrt{2} \text{ のとき, } y = -2a = 4 + 2\sqrt{2}$$

$$\mathbf{【9】} (1) \quad \frac{x - 1}{\sqrt{2}} = \frac{\sqrt{2}x + 5}{4} \text{ の両辺に } 4\sqrt{2} \text{ をかけて,}$$

$$4(x - 1) = \sqrt{2}(\sqrt{2}x + 5)$$

$$4x - 4 = 2x + 5\sqrt{2}$$

$$2x = 5\sqrt{2} + 4$$

$$x = \frac{5\sqrt{2} + 4}{2}$$

$$(2) \begin{cases} \sqrt{2}x + \sqrt{3}y = 1 \cdots \cdots \textcircled{1} \\ \sqrt{3}x - \sqrt{2}y = 1 \cdots \cdots \textcircled{2} \end{cases}$$

$$\textcircled{1} \times \sqrt{3} - \textcircled{2} \times \sqrt{2} \text{ より,}$$

$$y = \frac{\sqrt{3} - \sqrt{2}}{5}$$

$$\textcircled{1} \times \sqrt{2} + \textcircled{2} \times \sqrt{3} \text{ より,}$$

$$x = \frac{\sqrt{2} + \sqrt{3}}{5}$$

$$\text{よって, } x = \frac{\sqrt{2} + \sqrt{3}}{5}, y = \frac{\sqrt{3} - \sqrt{2}}{5}$$

$$\begin{aligned} \text{【10】 (1)} \quad & \frac{1}{1 + \sqrt{2} + \sqrt{3}} \\ &= \frac{1 + \sqrt{2} - \sqrt{3}}{(1 + \sqrt{2} + \sqrt{3})(1 + \sqrt{2} - \sqrt{3})} \\ &= \frac{1 + \sqrt{2} - \sqrt{3}}{1 + 2\sqrt{2} + 2 - 3} \\ &= \frac{1 + \sqrt{2} - \sqrt{3}}{2\sqrt{2}} \\ &= \frac{(1 + \sqrt{2} - \sqrt{3})\sqrt{2}}{2\sqrt{2} \times \sqrt{2}} \\ &= \frac{\sqrt{2} + 2 - \sqrt{6}}{4} \end{aligned}$$

$$\begin{aligned} (2) \quad & \frac{12}{\sqrt{2} - \sqrt{3} + \sqrt{5}} \\ &= \frac{12(\sqrt{2} - \sqrt{3} - \sqrt{5})}{(\sqrt{2} - \sqrt{3} + \sqrt{5})(\sqrt{2} - \sqrt{3} - \sqrt{5})} \\ &= \frac{12(\sqrt{2} - \sqrt{3} - \sqrt{5})}{(\sqrt{2} - \sqrt{3})^2 - 5} \\ &= \frac{12(\sqrt{2} - \sqrt{3} - \sqrt{5})}{2 - 2\sqrt{6} + 3 - 5} \\ &= \frac{12(\sqrt{2} - \sqrt{3} - \sqrt{5})}{-2\sqrt{6}} \\ &= \frac{12(\sqrt{2} - \sqrt{3} - \sqrt{5})\sqrt{6}}{-2\sqrt{6} \times \sqrt{6}} \\ &= \frac{12(\sqrt{12} - \sqrt{18} - \sqrt{30})}{-12} \\ &= -2\sqrt{3} + 3\sqrt{2} + \sqrt{30} \end{aligned}$$

$$\begin{aligned}
 \text{【11】 } x &= \frac{3}{\sqrt{7}-2} = \frac{3(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)} \\
 &= \frac{3(\sqrt{7}+2)}{7-4} \\
 &= \sqrt{7}+2
 \end{aligned}$$

$$\begin{aligned}
 y &= \frac{3}{\sqrt{7}+2} = \frac{3(\sqrt{7}-2)}{(\sqrt{7}+2)(\sqrt{7}-2)} \\
 &= \frac{3(\sqrt{7}-2)}{7-4} \\
 &= \sqrt{7}-2
 \end{aligned}$$

より, $x+y=2\sqrt{7}$, $xy=7-4=3$, $x-y=4$

$$\begin{array}{ll}
 (1) & x^2+y^2 \\
 & =(x+y)^2-2xy \\
 & =(2\sqrt{7})^2-2 \times 3 \\
 & =28-6 \\
 & =\mathbf{22}
 \end{array}
 \qquad
 \begin{array}{ll}
 (2) & 2x^2+3xy+2y^2 \\
 & =2(x+y)^2-xy \\
 & =2(2\sqrt{7})^2-3 \\
 & =56-3 \\
 & =\mathbf{53}
 \end{array}$$

$$\begin{array}{ll}
 (3) & \frac{1}{x}+\frac{1}{y} \\
 & =\frac{x+y}{xy} \\
 & =\frac{\mathbf{2\sqrt{7}}}{\mathbf{3}}
 \end{array}
 \qquad
 \begin{array}{ll}
 (4) & \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} \\
 & =\frac{(\sqrt{x}+\sqrt{y})^2}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})} \\
 & =\frac{x+2\sqrt{xy}+y}{x-y} \\
 & =\frac{2\sqrt{7}+2\sqrt{3}}{4} \\
 & =\frac{\sqrt{7}+\sqrt{3}}{2}
 \end{array}$$

【12】 (1) $(\sqrt{a}+\sqrt{b})^2 = a + b + 2\sqrt{ab}$

(2) ① $\sqrt{3+2\sqrt{2}} = \sqrt{1+2+2\sqrt{1 \times 2}}$ とみると, ルートの中は (1) の右辺の $a=1$, $b=2$ としたケースであると考えられる. よって,

$$\begin{aligned}
 \sqrt{3+2\sqrt{2}} &= \sqrt{(\sqrt{1}+\sqrt{2})^2} \\
 &= 1+\sqrt{2} \\
 &\doteq 1+1.41 = \mathbf{2.41}
 \end{aligned}$$

$$\begin{aligned}
 \text{② } \sqrt{5+2\sqrt{6}} &= \sqrt{2+3+2\sqrt{2 \times 3}} \\
 &= \sqrt{(\sqrt{2}+\sqrt{3})^2} \\
 &= \sqrt{2}+\sqrt{3} \\
 &\doteq 1.41+1.73 \\
 &= \mathbf{3.14}
 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad \sqrt{11 - 2\sqrt{24}} &= \sqrt{8 + 3 - 2\sqrt{8 \times 3}} \\ \text{ここで } (\sqrt{a} - \sqrt{b})^2 &= a + b - 2\sqrt{ab} \text{ となることから,} \\ \sqrt{8 + 3 - 2\sqrt{8 \times 3}} &= \sqrt{(\sqrt{8} - \sqrt{3})^2} \\ &= \sqrt{8} - \sqrt{3} \\ &= 2\sqrt{2} - \sqrt{3} \\ &\doteq 2.82 - 1.73 \\ &= \mathbf{1.09} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad \sqrt{21 + 6\sqrt{6}} &= \sqrt{21 + 2\sqrt{54}} \\ &= \sqrt{(\sqrt{18} + \sqrt{3})^2} \\ &= \sqrt{18} + \sqrt{3} \\ &= 3\sqrt{2} + \sqrt{3} \\ &\doteq 4.23 + 1.73 \\ &= \mathbf{5.96} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad \sqrt{2 + \sqrt{3}} &= \sqrt{\frac{4 + 2\sqrt{3}}{2}} \\ &= \frac{\sqrt{4 + 2\sqrt{3}}}{\sqrt{2}} \\ &= \frac{\sqrt{3} + \sqrt{1}}{\sqrt{2}} \\ &= \frac{(\sqrt{3} + 1)\sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\ &= \frac{(\sqrt{3} + 1)\sqrt{2}}{2} \\ &\doteq \frac{(1.73 + 1) \times 1.41}{2} \\ &= 1.92465 \\ &\doteq \mathbf{1.92} \end{aligned}$$

添削課題

$$\begin{aligned} \text{【1】 (1)} \quad & (3\sqrt{2} - 2)^2 \\ & = (3\sqrt{2})^2 - 2 \times 3\sqrt{2} \times 2 + 2^2 \\ & = 9 \times 2 - 12\sqrt{2} + 4 \\ & = \mathbf{22 - 12\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \text{(2)} \quad & (\sqrt{2} + 2)^2 - 2\sqrt{8} \\ & = (\sqrt{2})^2 + 2 \times \sqrt{2} \times 2 + 4 - 2 \times 2\sqrt{2} \\ & = \mathbf{6} \end{aligned}$$

$$\begin{aligned} \text{(3)} \quad & (\sqrt{18} + 3\sqrt{5})(3\sqrt{2} - \sqrt{45}) = (3\sqrt{2} + 3\sqrt{5})(3\sqrt{2} - 3\sqrt{5}) \\ & = (3\sqrt{2})^2 - (3\sqrt{5})^2 \\ & = 18 - 45 \\ & = \mathbf{-27} \end{aligned}$$

$$\begin{aligned} \text{(4)} \quad & (\sqrt{2} - \sqrt{3})^2 + \sqrt{\frac{3}{2}} - \frac{2}{\sqrt{6}} \\ & = (\sqrt{2})^2 - 2\sqrt{2} \times \sqrt{3} + (\sqrt{3})^2 + \frac{\sqrt{3}}{\sqrt{2}} - \frac{2\sqrt{6}}{\sqrt{6} \times \sqrt{6}} \\ & = 2 - 2\sqrt{6} + 3 + \frac{\sqrt{6}}{2} - \frac{2\sqrt{6}}{6} \\ & = 5 + \frac{-12 + 3 - 2}{6} \sqrt{6} \\ & = \mathbf{5 - \frac{11\sqrt{6}}{6}} \end{aligned}$$

$$\begin{aligned} \text{(5)} \quad & \sqrt{5} + 1 = A, \quad \sqrt{5} - 1 = B \quad \text{とおくと,} \\ & (\sqrt{5} + 1)^2 - (\sqrt{5} - 1)^2 = A^2 - B^2 = (A + B)(A - B) \\ & = \{(\sqrt{5} + 1) + (\sqrt{5} - 1)\} \times \{(\sqrt{5} + 1) - (\sqrt{5} - 1)\} \\ & = 2\sqrt{5} \times 2 \\ & = \mathbf{4\sqrt{5}} \end{aligned}$$

$$\begin{aligned} \text{(6)} \quad & 2\sqrt{7} + 1 = A \quad \text{とおくと,} \\ & (2\sqrt{7} + 1)^2 - 2(2\sqrt{7} + 1) + 1 = A^2 - 2A + 1 \\ & = (A - 1)^2 \\ & = (2\sqrt{7})^2 \\ & = \mathbf{28} \end{aligned}$$

(7) $\sqrt{6} + 2 = A$ とおくと,

$$\begin{aligned}(\sqrt{6} + 2)^2 - 8(\sqrt{6} + 2) + 16 &= A^2 - 8A + 16 = (A - 4)^2 \\ &= (\sqrt{6} + 2 - 4)^2 = (\sqrt{6} - 2)^2 \\ &= 6 - 4\sqrt{6} + 4 = \mathbf{10 - 4\sqrt{6}}\end{aligned}$$

(8) $\sqrt{3} + 8 = A$ とおくと,

$$\begin{aligned}(\sqrt{3} + 8)^2 - 11(\sqrt{3} + 8) + 24 &= A^2 - 11A + 24 = (A - 3)(A - 8) \\ &= (\sqrt{3} + 8 - 3)(\sqrt{3} + 8 - 8) \\ &= (\sqrt{3} + 5) \times \sqrt{3} \\ &= \mathbf{3 + 5\sqrt{3}}\end{aligned}$$

[2] (1) $\frac{2}{\sqrt{2}} = \sqrt{2}$
 $= \mathbf{1.414}$

(2) $\frac{1}{\sqrt{3} - \sqrt{2}} = \frac{\sqrt{3} + \sqrt{2}}{3 - 2}$
 $= 1.414 + 1.732$
 $= \mathbf{3.146}$

(3) $\frac{2 + \sqrt{3}}{2 - \sqrt{3}} = \frac{(2 + \sqrt{3})^2}{(2 - \sqrt{3})(2 + \sqrt{3})}$
 $= \frac{4 + 4\sqrt{3} + 3}{4 - 3}$
 $= 7 + 4\sqrt{3}$
 $= \mathbf{13.928}$

(4) $\frac{5(\sqrt{2} + \sqrt{3})}{\sqrt{8} - \sqrt{3}} + \frac{6\sqrt{2}}{1 - \sqrt{3}}$
 $= \frac{5(\sqrt{2} + \sqrt{3})(2\sqrt{2} + \sqrt{3})}{(2\sqrt{2} - \sqrt{3})(2\sqrt{2} + \sqrt{3})} + \frac{6\sqrt{2}(1 + \sqrt{3})}{1 - 3}$
 $= \frac{5(4 + 3\sqrt{6} + 3)}{8 - 3} + \frac{6\sqrt{2} + 6\sqrt{6}}{-2}$
 $= 7 + 3\sqrt{6} - 3\sqrt{2} - 3\sqrt{6}$
 $= 7 - 3\sqrt{2}$
 $= \mathbf{2.758}$

[3] (1) ① $\sqrt{4} < \sqrt{7} < \sqrt{9}$ より, $2 < \sqrt{7} < 3$ $\therefore a = 2$

② $a + b = \sqrt{7}$ より, $b = \sqrt{7} - a = \sqrt{7} - 2$

③ $b^2 + 2ab = b(b + 2a) = (\sqrt{7} - 2)(\sqrt{7} - 2 + 4)$
 $= (\sqrt{7} - 2)(\sqrt{7} + 2)$
 $= 7 - 4$
 $= 3$

(2) ① $\sqrt{16} < 3\sqrt{2} = \sqrt{18} < \sqrt{25}$ より, $4 < 3\sqrt{2} < 5$ $\therefore a = 4$

② $a + b = 3\sqrt{2}$ より, $b = 3\sqrt{2} - a = 3\sqrt{2} - 4$

③ $\frac{a}{b+1} + \frac{b}{3} = \frac{4}{3\sqrt{2}-3} + \frac{3\sqrt{2}-4}{3}$
 $= \frac{4(\sqrt{2}+1)}{3(\sqrt{2}-1)(\sqrt{2}+1)} + \frac{3\sqrt{2}-4}{3}$
 $= \frac{4\sqrt{2}+4}{3} + \frac{3\sqrt{2}-4}{3}$
 $= \frac{7\sqrt{2}}{3}$

[4] (1) $x + y = \frac{1}{\sqrt{5}+2} + \frac{1}{\sqrt{5}-2}$
 $= \frac{\sqrt{5}-2 + \sqrt{5}+2}{(\sqrt{5}+2)(\sqrt{5}-2)}$
 $= 2\sqrt{5}$

(2) $xy = \frac{1}{\sqrt{5}+2} \times \frac{1}{\sqrt{5}-2}$
 $= \frac{1}{5-4}$
 $= 1$

(3) $x^2 + y^2 = (x+y)^2 - 2xy$
 $= (2\sqrt{5})^2 - 2 \times 1$
 $= 20 - 2$
 $= 18$

(4) $\frac{y}{x} + \frac{x}{y} = \frac{y^2 + x^2}{xy}$
 $= \frac{18}{1}$
 $= 18$

(5) $x^4 + y^4 = (x^2 + y^2)^2 - 2x^2y^2$
 $= (18^2 - 2)^2 - 2(xy)^2$
 $= 324 - 2$
 $= 322$

小テスト

- 【1】** (1) $11\sqrt{2}$
(2) **6**
(3) $\sqrt{3}$
(4) $14 + 8\sqrt{3}$
(5) **23**

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会員番号	
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氏名	
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