3MJSS/3MJS/3MJ

本科２期11月度

解答

Ｚ会東大進学教室

中３選抜東大・医学部数学

中３数学

中３東大数学

２１章　三角比（１）－鋭角－

問題

【1】(1)

$$\sin\alpha = \frac{BC}{AB} = \frac{24}{26} = \mathbf{\frac{12}{13}}$$
$$\cos\alpha = \frac{AC}{AB} = \frac{10}{26} = \mathbf{\frac{5}{13}}$$
$$\tan\alpha = \frac{BC}{AC} = \frac{24}{10} = \mathbf{\frac{12}{5}}$$

(2)

$$\sin\alpha = \frac{AC}{AB} = \frac{6}{9} = \mathbf{\frac{2}{3}}$$
$$\cos\alpha = \frac{BC}{AB} = \frac{3\sqrt{5}}{9} = \mathbf{\frac{\sqrt{5}}{3}}$$
$$\tan\alpha = \frac{AC}{BC} = \frac{6}{3\sqrt{5}} = \mathbf{\frac{2\sqrt{5}}{5}}$$

(3)

$$\sin\alpha = \frac{AB}{BC} = \frac{12}{4\sqrt{13}} = \mathbf{\frac{3\sqrt{13}}{13}}$$
$$\cos\alpha = \frac{AC}{BC} = \frac{8}{4\sqrt{13}} = \mathbf{\frac{2\sqrt{13}}{13}}$$
$$\tan\alpha = \frac{AB}{AC} = \frac{12}{8} = \mathbf{\frac{3}{2}}$$

(4)

$$\sin\alpha = \frac{BC}{AB} = \frac{\sqrt{3}}{\sqrt{7}} = \mathbf{\frac{\sqrt{21}}{7}}$$
$$\cos\alpha = \frac{AC}{AB} = \frac{2}{\sqrt{7}} = \mathbf{\frac{2\sqrt{7}}{7}}$$
$$\tan\alpha = \frac{BC}{AC} = \mathbf{\frac{\sqrt{3}}{2}}$$

【2】(1) 三平方の定理より

$$BC = \sqrt{5^2+4^2} = \sqrt{25+16}$$
$$= \sqrt{41}$$

なので

$$\sin\alpha = \frac{AC}{BC} = \frac{4}{\sqrt{41}} = \mathbf{\frac{4\sqrt{41}}{41}}$$
$$\cos\alpha = \frac{AB}{BC} = \frac{5}{\sqrt{41}} = \mathbf{\frac{5\sqrt{41}}{41}}$$
$$\tan\alpha = \frac{AC}{AB} = \mathbf{\frac{4}{5}}$$

(2) 三平方の定理より

$$AC = \sqrt{7^2-5^2} = \sqrt{49-25}$$
$$= \sqrt{24} = 2\sqrt{6}$$

なので

$$\sin\alpha = \frac{AB}{BC} = \mathbf{\frac{5}{7}}$$
$$\cos\alpha = \frac{AC}{BC} = \mathbf{\frac{2\sqrt{6}}{7}}$$
$$\tan\alpha = \frac{AB}{AC} = \frac{5}{2\sqrt{6}} = \mathbf{\frac{5\sqrt{6}}{12}}$$

(3) 三平方の定理より

$$BC = \sqrt{10^2-5^2}$$
$$= \sqrt{100-25}$$
$$= \sqrt{75} = 5\sqrt{3}$$

なので

$$\sin\alpha = \frac{BC}{AC} = \frac{5\sqrt{3}}{10} = \mathbf{\frac{\sqrt{3}}{2}}$$
$$\cos\alpha = \frac{AB}{AC} = \frac{5}{10} = \mathbf{\frac{1}{2}}$$
$$\tan\alpha = \frac{BC}{AB} = \frac{5\sqrt{3}}{5} = \mathbf{\sqrt{3}}$$

(4) 三平方の定理より

$$BC = \sqrt{4^2-(2\sqrt{2})^2}$$
$$= \sqrt{16-8}$$
$$= \sqrt{8} = 2\sqrt{2}$$

なので

$$\sin\alpha = \frac{AB}{AC} = \frac{2\sqrt{2}}{4} = \mathbf{\frac{\sqrt{2}}{2}}$$
$$\cos\alpha = \frac{BC}{AC} = \frac{2\sqrt{2}}{4} = \mathbf{\frac{\sqrt{2}}{2}}$$
$$\tan\alpha = \frac{AB}{BC} = \frac{2\sqrt{2}}{2\sqrt{2}} = \mathbf{1}$$

【3】(1)

$$\begin{aligned}\sin 60^\circ - \cos 30^\circ + \tan 45^\circ &= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + 1 \\ &= \mathbf{1}\end{aligned}$$

(2)

$$\begin{aligned}\sin 30^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ &= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} \\ &= \boldsymbol{\frac{\sqrt{2}}{2}}\end{aligned}$$

(3)

$$\begin{aligned}\frac{\sin 60^\circ + \cos 30^\circ + \tan 60^\circ}{\sin 45^\circ + \cos 60^\circ - \tan 45^\circ} &= \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \sqrt{3}}{\frac{\sqrt{2}}{2} + \frac{1}{2} - 1} \\ &= \frac{2\sqrt{3}}{\frac{\sqrt{2}}{2} - \frac{1}{2}} \\ &= \frac{2\sqrt{3}}{\frac{\sqrt{2}-1}{2}} = \frac{4\sqrt{3}}{\sqrt{2}-1} = 4\sqrt{3}(\sqrt{2}+1) \\ &= \mathbf{4\sqrt{6} + 4\sqrt{3}}\end{aligned}$$

(4)

$$\begin{aligned}(\sin 30^\circ + \cos 30^\circ)(\sin 45^\circ - \cos 60^\circ) &= \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right) \\ &= \frac{1+\sqrt{3}}{2} \times \frac{\sqrt{2}-1}{2} \\ &= \frac{\sqrt{2} - 1 + \sqrt{6} - \sqrt{3}}{4} \\ &= \boldsymbol{\frac{\sqrt{6} - \sqrt{3} + \sqrt{2} - 1}{4}}\end{aligned}$$

【4】(1)

$$\begin{aligned}\frac{\mathrm{BC}}{\mathrm{AB}} &= \cos\alpha \\ \frac{\mathrm{BC}}{1} &= \cos\alpha \\ \mathrm{BC} &= \boldsymbol{\cos\alpha}\end{aligned}$$

また

$$\begin{aligned}\frac{\mathrm{CA}}{\mathrm{AB}} &= \sin\alpha \\ \frac{\mathrm{CA}}{1} &= \sin\alpha \\ \mathrm{CA} &= \boldsymbol{\sin\alpha}\end{aligned}$$

(2)

$$\begin{aligned}\frac{\mathrm{BC}}{\mathrm{AB}} &= \sin(90^\circ - \alpha) \\ \frac{\mathrm{BC}}{1} &= \sin(90^\circ - \alpha) \\ \mathrm{BC} &= \boldsymbol{\sin(90^\circ - \alpha)}\end{aligned}$$

また

$$\begin{aligned}\frac{\mathrm{CA}}{\mathrm{AB}} &= \cos(90^\circ - \alpha) \\ \frac{\mathrm{CA}}{1} &= \cos(90^\circ - \alpha) \\ \mathrm{CA} &= \boldsymbol{\cos(90^\circ - \alpha)}\end{aligned}$$

(3) (1), (2) より $\boldsymbol{\sin\alpha = \cos(90^\circ - \alpha)}$

$\boldsymbol{\cos\alpha = \sin(90^\circ - \alpha)}$

【5】(1)

$$\frac{\mathrm{AH}}{\mathrm{AB}} = \sin(\alpha+\beta)$$
$$\frac{\mathrm{AH}}{1} = \sin(\alpha+\beta)$$
$$\mathrm{AH} = \boldsymbol{\sin(\alpha+\beta)}$$

(2)

$$\frac{\mathrm{BE}}{\mathrm{AB}} = \cos\beta$$
$$\frac{\mathrm{BE}}{1} = \cos\beta$$
$$\mathrm{BE} = \boldsymbol{\cos\beta}$$

また

$$\frac{\mathrm{AE}}{\mathrm{AB}} = \sin\beta$$
$$\frac{\mathrm{AE}}{1} = \sin\beta$$
$$\mathrm{AE} = \boldsymbol{\sin\beta}$$

図 1

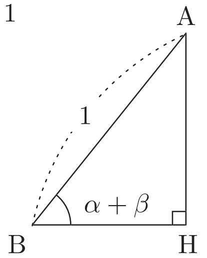


図 2

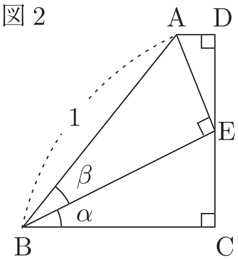
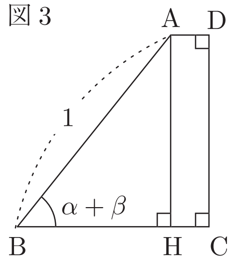


図 3



(3)

$$\frac{\mathrm{CE}}{\mathrm{BE}} = \sin\alpha$$
$$\frac{\mathrm{CE}}{\cos\beta} = \sin\alpha$$
$$\mathrm{CE} = \boldsymbol{\sin\alpha\cos\beta}$$

△BCE において,

$$\angle\mathrm{CEB} = 180^\circ - 90^\circ - \alpha = 90^\circ - \alpha$$

また,

$$\begin{aligned}\angle\mathrm{AED} &= 180^\circ - \angle\mathrm{AEC}\\ &= 180^\circ - (\angle\mathrm{AEB} + \angle\mathrm{CEB})\\ &= 180^\circ - \{90^\circ + (90^\circ - \alpha)\} = \alpha\end{aligned}$$

よって

$$\frac{\mathrm{ED}}{\mathrm{AE}} = \cos\alpha$$
$$\frac{\mathrm{ED}}{\sin\beta} = \cos\alpha$$
$$\mathrm{ED} = \boldsymbol{\cos\alpha\sin\beta}$$

(4) 図 3 より

$$\mathrm{AH} = \mathrm{CD}$$
$$\mathrm{AH} = \mathrm{CE} + \mathrm{ED}$$
$$\boldsymbol{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta}$$

(5)

$$\frac{\mathrm{BC}}{\mathrm{BE}} = \cos\alpha$$
$$\frac{\mathrm{BC}}{\cos\beta} = \cos\alpha$$
$$\mathrm{BC} = \boldsymbol{\cos\alpha\cos\beta}$$

また

$$\frac{\mathrm{AD}}{\mathrm{AE}} = \sin\alpha$$
$$\frac{\mathrm{AD}}{\sin\beta} = \sin\alpha$$
$$\mathrm{AD} = \boldsymbol{\sin\alpha\sin\beta}$$

(6) $\mathrm{AH} = \mathrm{CD}$ が成り立つとき,

図 1 より, $\mathrm{BH} = \cos(\alpha+\beta)$

(5) より,

$\mathrm{BC} = \cos\alpha\cos\beta$

$\mathrm{AD} = \sin\alpha\sin\beta$

図 3 より

$\mathrm{BH} = \mathrm{BC} - \mathrm{HC} = \mathrm{BC} - \mathrm{AD}$

よって

$\boldsymbol{\cos(\alpha+\beta)}$

$\boldsymbol{= \cos\alpha\cos\beta - \sin\alpha\sin\beta}$

【6】(1)

$$\frac{\mathrm{AB}}{\mathrm{BC}} = \cos 30^\circ$$
$$\frac{x}{1} = \frac{\sqrt{3}}{2}$$
$$x = \boldsymbol{\frac{\sqrt{3}}{2}}$$
$$\frac{\mathrm{AC}}{\mathrm{BC}} = \sin 30^\circ$$
$$\frac{y}{1} = \frac{1}{2}$$
$$y = \boldsymbol{\frac{1}{2}}$$

A から BC に下した垂線の足を H とすると,

$$\frac{\mathrm{AH}}{\mathrm{AB}} = \sin 30^\circ$$
$$\frac{z}{\frac{\sqrt{3}}{2}} = \frac{1}{2}$$
$$z = \boldsymbol{\frac{\sqrt{3}}{4}}$$

(2)

$$\frac{\mathrm{OB}}{\mathrm{OA}} = \cos 30^\circ$$
$$\frac{x}{a} = \frac{\sqrt{3}}{2}$$
$$x = \boldsymbol{\frac{\sqrt{3}}{2}a}$$
$$\frac{\mathrm{OC}}{\mathrm{OB}} = \cos 30^\circ$$
$$\frac{y}{x} = \frac{\sqrt{3}}{2}$$
$$y = \frac{\sqrt{3}}{2}x$$

$x = \frac{\sqrt{3}}{2}a$ を代入して

$$y = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}a = \boldsymbol{\frac{3}{4}a}$$
$$\frac{\mathrm{OD}}{\mathrm{OC}} = \cos 30^\circ$$
$$\frac{z}{y} = \frac{\sqrt{3}}{2}$$
$$z = \frac{\sqrt{3}}{2}y$$

$y = \frac{3}{4}a$ を代入して

$$z = \frac{\sqrt{3}}{2} \times \frac{3}{4}a = \boldsymbol{\frac{3\sqrt{3}}{8}a}$$

(3)

$$\frac{\mathrm{OB}}{\mathrm{OA}} = \sin 60^\circ$$
$$\frac{x}{a} = \frac{\sqrt{3}}{2}$$
$$x = \boldsymbol{\frac{\sqrt{3}}{2}a}$$
$$\frac{\mathrm{OC}}{\mathrm{OB}} = \sin 60^\circ$$
$$\frac{y}{x} = \frac{\sqrt{3}}{2}$$
$$y = \frac{\sqrt{3}}{2}x$$

$x = \frac{\sqrt{3}}{2}a$ を代入して

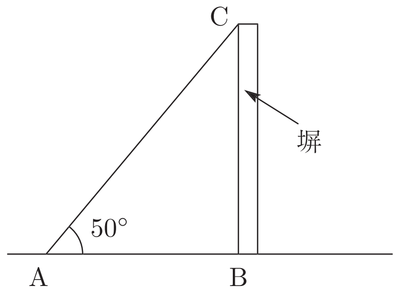
$$y = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}a = \boldsymbol{\frac{3}{4}a}$$
$$\frac{\mathrm{OD}}{\mathrm{OC}} = \sin 60^\circ$$
$$\frac{z}{y} = \frac{\sqrt{3}}{2}$$
$$z = \frac{\sqrt{3}}{2}y$$

$y = \frac{3}{4}a$ を代入して

$$z = \frac{\sqrt{3}}{2} \times \frac{3}{4}a = \boldsymbol{\frac{3\sqrt{3}}{8}a}$$

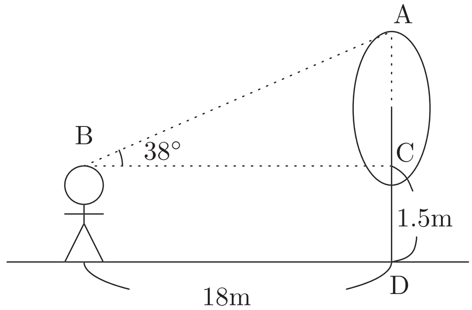
【7】図のように A, B, C を定めると

$$\frac{BC}{AC} = \sin 50^\circ$$
$$\frac{BC}{5} = 0.7660$$
$$BC = 5 \times 0.7660$$
$$= \mathbf{3.83(m)}$$



【8】図のように A, B, C, D を定めると

$$\frac{AC}{BC} = \tan 38^\circ$$
$$\frac{AC}{18} = 0.7813$$
$$AC = 18 \times 0.7813$$
$$= 14.0634$$

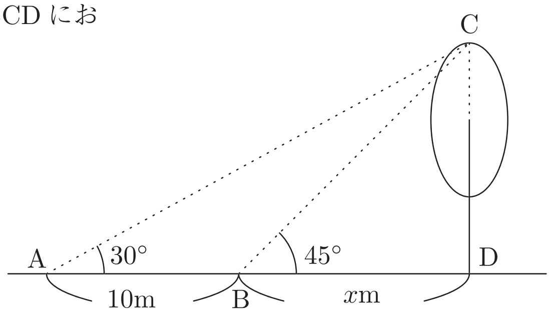


木の高さは

$$AD = AC + CD$$
$$= 14.0634 + 1.5$$
$$= \mathbf{15.5634(m)}$$

【9】木の高さ CD を x (m) とおくと，△BCD において

$$\frac{CD}{BD} = \tan 45^\circ$$
$$\frac{x}{BD} = 1$$
$$BD = x$$



また，△ADC において

$$\frac{CD}{AD} = \tan 30^\circ$$
$$\frac{x}{10+x} = \frac{1}{\sqrt{3}}$$
$$(\sqrt{3}-1)x = 10$$
$$x = \frac{10}{\sqrt{3}-1} = \frac{10(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$
$$= 5\sqrt{3} + 5$$
$$= 5 \times 1.732 + 5 \ (\sqrt{3} = 1.732 \text{ とした})$$
$$= 13.66$$

よって，**13.66(m)**

【10】(1) $\angle DAC = 45°$，$\angle BAC = 60°$ より，
$\triangle ADC$ において

$$\frac{CD}{AC} = \tan 45°$$
$$\frac{CD}{1} = 1$$
$$CD = 1$$

$\triangle ABC$ において

$$\frac{BC}{AC} = \tan 60°$$
$$\frac{BC}{1} = \sqrt{3}$$
$$BC = \sqrt{3}$$

よって，

$$BD = BC - CD$$
$$= \boldsymbol{\sqrt{3} - 1}$$

(2) D から AB におろした垂線の足を H とする．$\triangle BDH$ において

$$\frac{DH}{BD} = \sin 30°$$
$$\frac{DH}{\sqrt{3}-1} = \frac{1}{2} \quad \therefore\ DH = \frac{\sqrt{3}-1}{2}$$

$\triangle ACD$ において

$$\frac{AC}{AD} = \sin 45°$$
$$\frac{1}{AD} = \frac{1}{\sqrt{2}}$$
$$AD = \sqrt{2}$$

$\triangle ADH$ において，$\angle DAH = 15°$ より

$$\sin 15° = \frac{DH}{AD} = \frac{\frac{\sqrt{3}-1}{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$
$$= \boldsymbol{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

【11】(1) $\triangle ABC$ において，

$$BC = \sqrt{3}$$

なので，

$$BD = BC - CD$$
$$= \sqrt{3} - 1$$

$\triangle BDE$ は $\angle EBD = \angle BDE = 45°$ より，

$$\frac{BE}{BD} = \sin 45°$$
$$\frac{BE}{\sqrt{3}-1} = \frac{1}{\sqrt{2}}$$
$$BE = \frac{\sqrt{6}-\sqrt{2}}{2}$$

また，$BE = ED$ より，

$$ED = \frac{\sqrt{6}-\sqrt{2}}{2}$$

$\triangle ACD$ において，

$$AD = \sqrt{2}$$

より，

$$AE = AD + DE$$
$$= \sqrt{2} + \frac{\sqrt{6}-\sqrt{2}}{2}$$
$$= \frac{\sqrt{6}+\sqrt{2}}{2}$$

$\triangle ABE$ において，

$$\sin 15° = \frac{BE}{AB}$$
$$= \frac{\frac{\sqrt{6}-\sqrt{2}}{2}}{2}$$
$$= \boldsymbol{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

(2)

$$\cos 15^\circ = \frac{\mathrm{AE}}{\mathrm{AB}} = \frac{\frac{\sqrt{6}+\sqrt{2}}{2}}{2} = \boldsymbol{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

(3)

$$\tan 15^\circ = \frac{\mathrm{BE}}{\mathrm{AE}} = \frac{\frac{\sqrt{6}-\sqrt{2}}{2}}{\frac{\sqrt{6}+\sqrt{2}}{2}} = \frac{8-4\sqrt{3}}{4} = \boldsymbol{2-\sqrt{3}}$$

(4) △ABE において，∠ABE = 75° だから

$$\sin 75^\circ = \frac{\mathrm{AE}}{\mathrm{AB}} = \frac{\frac{\sqrt{6}+\sqrt{2}}{2}}{2} = \boldsymbol{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

(5)

$$\cos 75^\circ = \frac{\mathrm{BE}}{\mathrm{AB}} = \frac{\frac{\sqrt{6}-\sqrt{2}}{2}}{2} = \boldsymbol{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

(6)

$$\tan 75^\circ = \frac{\mathrm{AE}}{\mathrm{BE}} = \frac{\frac{\sqrt{6}+\sqrt{2}}{2}}{\frac{\sqrt{6}-\sqrt{2}}{2}} = \frac{8+4\sqrt{3}}{4} = \boldsymbol{2+\sqrt{3}}$$

【12】(1) △ABC において

$$\angle\mathrm{ABC} = 72^\circ$$

よって

$$\angle\mathrm{ABD} = \angle\mathrm{DBC} = 36^\circ$$

これより，△ABD，△BCD は二等辺三角形である．

$$\mathrm{BC} = \mathrm{BD} = \mathrm{AD} = 1$$

ここで，CD $= x$ とおく．
△ABC ∽ △BCD であるので

$$\begin{aligned} \mathrm{AC} : \mathrm{BD} &= \mathrm{BC} : \mathrm{CD} \\ (1+x) : 1 &= 1 : x \\ x(1+x) &= 1 \\ x^2 + x - 1 &= 0 \\ x &= \frac{-1 \pm \sqrt{1+4}}{2} \\ x &= \frac{-1 \pm \sqrt{5}}{2} \end{aligned}$$

$x > 0$ より

$$x = \frac{-1+\sqrt{5}}{2}$$

よって

$$\mathrm{CD} = \boldsymbol{\frac{-1+\sqrt{5}}{2}}$$

また

$$\begin{aligned} \mathrm{AB} &= \mathrm{AD} + \mathrm{DC} \\ &= 1 + \frac{-1+\sqrt{5}}{2} \\ &= \boldsymbol{\frac{1+\sqrt{5}}{2}} \end{aligned}$$

(2) A から BC におろした垂線の足を H とすると

$$\begin{aligned} \angle\mathrm{BAH} &= 36^\circ \times \frac{1}{2} = 18^\circ \\ \mathrm{BH} &= 1 \times \frac{1}{2} = \frac{1}{2} \end{aligned}$$

より，△ABH において

$$\begin{aligned} \sin 18^\circ &= \frac{\mathrm{BH}}{\mathrm{AB}} \\ &= \frac{\frac{1}{2}}{\frac{1+\sqrt{5}}{2}} \\ &= \frac{1}{1+\sqrt{5}} = \boldsymbol{\frac{\sqrt{5}-1}{4}} \end{aligned}$$

D から AB におろした垂線の足を I とすると

$$\mathrm{BI} = \frac{1+\sqrt{5}}{2} \times \frac{1}{2} = \frac{1+\sqrt{5}}{4}$$

より，△BDI において

$$\begin{aligned} \cos 36^\circ &= \frac{\mathrm{BI}}{\mathrm{BD}} \\ &= \frac{\frac{1+\sqrt{5}}{4}}{1} = \boldsymbol{\frac{1+\sqrt{5}}{4}} \end{aligned}$$

【13】(1)

$$\angle \mathrm{BAC} = 108^\circ$$
$$\angle \mathrm{BAD} = \angle \mathrm{BAC} - \angle \mathrm{DAC} = 108^\circ - 36^\circ = 72^\circ$$
$$\angle \mathrm{BDA} = 72^\circ$$

より，$\triangle$ABD は二等辺三角形なので

AB $=$ BD $= 1$

また，$\triangle \mathrm{ABC} \backsim \triangle \mathrm{DAC}$ なので，

CD $= x$ とおくと，

$$\mathrm{BC} : \mathrm{AC} = \mathrm{AC} : \mathrm{DC}$$
$$(1+x) : 1 = 1 : x$$
$$x(x+1) = 1$$
$$x^2 + x - 1 = 0$$
$$x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$x > 0$ より，$x = \dfrac{-1+\sqrt{5}}{2}$

よって

$$\mathrm{CD} = \boldsymbol{\frac{-1+\sqrt{5}}{2}}$$
$$\mathrm{BC} = \mathrm{BD} + \mathrm{CD}$$
$$= 1 + \frac{-1+\sqrt{5}}{2} = \boldsymbol{\frac{1+\sqrt{5}}{2}}$$

(2) A から BC におろした垂線の足を H とすると

$$\mathrm{BH} = \frac{1+\sqrt{5}}{2} \times \frac{1}{2} = \frac{1+\sqrt{5}}{4}$$

また，

$$\mathrm{AH}^2 = \mathrm{AB}^2 - \mathrm{BH}^2 = 1^2 - \left(\frac{1+\sqrt{5}}{4}\right)^2 = \frac{10-2\sqrt{5}}{16}$$
$$\mathrm{AH} = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

よって

$$\sin 36^\circ = \frac{\mathrm{AH}}{\mathrm{AB}} = \frac{\frac{\sqrt{10-2\sqrt{5}}}{4}}{1} = \boldsymbol{\frac{\sqrt{10-2\sqrt{5}}}{4}}$$

(3)

$$\cos 36^\circ = \frac{\mathrm{BH}}{\mathrm{AB}} = \frac{\frac{1+\sqrt{5}}{4}}{1} = \boldsymbol{\frac{1+\sqrt{5}}{4}}$$

(4)

$$\tan 36^\circ = \frac{\mathrm{AH}}{\mathrm{BH}} = \frac{\frac{\sqrt{10-2\sqrt{5}}}{4}}{\frac{1+\sqrt{5}}{4}} = \frac{\sqrt{10-2\sqrt{5}}}{1+\sqrt{5}} = \frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}$$
$$= \frac{\sqrt{10-2\sqrt{5}} \times \sqrt{(\sqrt{5}-1)^2}}{4} = \frac{\sqrt{10-2\sqrt{5}} \times \sqrt{6-2\sqrt{5}}}{4} = \frac{\sqrt{80-32\sqrt{5}}}{4} = \boldsymbol{\sqrt{5-2\sqrt{5}}}$$

(5) B から AD におろした垂線の足を I とすると

$$\begin{aligned}\mathrm{DI} &= \frac{-1+\sqrt{5}}{2} \times \frac{1}{2}\\ &= \frac{-1+\sqrt{5}}{4}\end{aligned}$$

また,

$$\begin{aligned}\mathrm{BI}^2 &= \mathrm{BD}^2 - \mathrm{DI}^2\\ &= 1^2 - \left(\frac{-1+\sqrt{5}}{4}\right)^2\\ &= \frac{10+2\sqrt{5}}{16}\\ \mathrm{BI} &= \frac{\sqrt{10+2\sqrt{5}}}{4}\end{aligned}$$

よって

$$\begin{aligned}\sin 72^\circ &= \frac{\mathrm{BI}}{\mathrm{BD}}\\ &= \frac{\frac{\sqrt{10+2\sqrt{5}}}{4}}{1} = \boldsymbol{\frac{\sqrt{10+2\sqrt{5}}}{4}}\end{aligned}$$

(6)

$$\begin{aligned}\cos 72^\circ &= \frac{\mathrm{DI}}{\mathrm{BD}}\\ &= \frac{\frac{-1+\sqrt{5}}{4}}{1} = \boldsymbol{\frac{-1+\sqrt{5}}{4}}\end{aligned}$$

(7)

$$\begin{aligned}\tan 72^\circ &= \frac{\mathrm{BI}}{\mathrm{DI}}\\ &= \frac{\frac{\sqrt{10+2\sqrt{5}}}{4}}{\frac{-1+\sqrt{5}}{4}}\\ &= \frac{\sqrt{10+2\sqrt{5}}}{-1+\sqrt{5}}\\ &= \frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}\\ &= \frac{\sqrt{10+2\sqrt{5}} \times \sqrt{(\sqrt{5}+1)^2}}{4}\\ &= \frac{\sqrt{10+2\sqrt{5}} \times \sqrt{6+2\sqrt{5}}}{4}\\ &= \frac{\sqrt{80+32\sqrt{5}}}{4} = \boldsymbol{\sqrt{5+2\sqrt{5}}}\end{aligned}$$

添削課題

【1】(1) 三平方の定理より，

$$AB^2 = 2^2 + 3^2 = 13$$

$\therefore$ $AB = \sqrt{13}$

よって，

$$\sin\theta = \frac{AC}{BA} = \frac{2}{\sqrt{13}} = \mathbf{\frac{2}{13}\sqrt{13}}$$

$$\cos\theta = \frac{BC}{AB} = \frac{3}{\sqrt{13}} = \mathbf{\frac{3}{13}\sqrt{13}}$$

$$\tan\theta = \frac{CA}{BC} = \mathbf{\frac{2}{3}}$$

(2) 三平方の定理より，

$$AC^2 = 25^2 - 24^2 = 49$$

$\therefore$ $AC = 7$

よって，

$$\sin\theta = \frac{BC}{AB} = \mathbf{\frac{24}{25}}$$

$$\cos\theta = \frac{AC}{BA} = \mathbf{\frac{7}{25}}$$

$$\tan\theta = \frac{CB}{AC} = \mathbf{\frac{24}{7}}$$

【2】$\angle B = 90° - 40° = 50°$ だから，

$BC = 10\sin 40° = 10 \times 0.64 = \mathbf{6.4}$

$CA = 10\sin 50° = 10 \times 0.77 = \mathbf{7.7}$

【3】(1)

$$(\text{与式}) = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \mathbf{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

(2)

$$(\text{与式}) = \frac{\frac{\sqrt{3}}{3}}{1+\sqrt{3}} = \frac{\sqrt{3}}{3}\cdot\frac{1}{1+\sqrt{3}} = \frac{\sqrt{3}}{3}\cdot\frac{1-\sqrt{3}}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{\sqrt{3}}{3}\cdot\frac{1-\sqrt{3}}{-2} = \mathbf{\frac{3-\sqrt{3}}{6}}$$

【4】右の図のように，P，Q，R を定める．

TR $= x$(m) とすると，△TBR は直角二等辺三角形であり，四角形 BQPR は長方形であるので，

$BR = QP = TR = x$(m)

また，

$RP = BQ = 150 - 100 = 50$(m)

よって，$\tan 30° = \frac{PT}{AP}$ より，

$$\frac{1}{\sqrt{3}} = \frac{50+x}{1000+x}$$

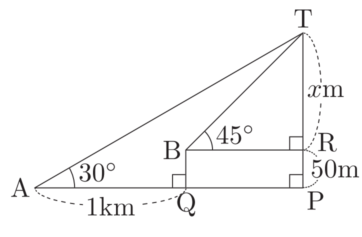
$$1000 + x = \sqrt{3}(50+x)$$

$$(\sqrt{3}-1)x = 1000 - 50\sqrt{3}$$

$$x = \frac{1000-50\sqrt{3}}{\sqrt{3}-1} = 475\sqrt{3} + 425$$

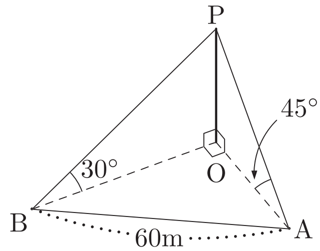
よって，標高は，

$x + 150 = \mathbf{475\sqrt{3} + 575}$



（答）**$475\sqrt{3} + 575$(m)**

【5】塔の高さを x(m) とすると，



$\triangle$POB において，

$$\frac{\mathrm{PO}}{\mathrm{BO}} = \frac{x}{\mathrm{BO}} = \tan 30^\circ$$

$\tan 30^\circ = \dfrac{1}{\sqrt{3}}$ より，

$$\mathrm{BO} = \sqrt{3}x$$

また，$\triangle$POA において，

$$\frac{\mathrm{PO}}{\mathrm{AO}} = \frac{x}{\mathrm{AO}} = \tan 45^\circ$$

$\tan 45^\circ = 1$ より，

$$\mathrm{AO} = x$$

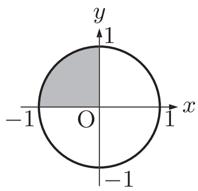
となる．ここで，$\triangle$AOB も直角三角形なので，三平方の定理より，

$$(\sqrt{3}x)^2 + x^2 = 60^2$$
$$4x^2 = 3600$$
$$x^2 = 900$$
$$x = \mathbf{30}$$

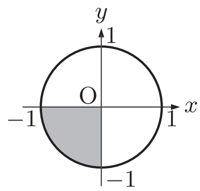
（答）**30 m**

問題

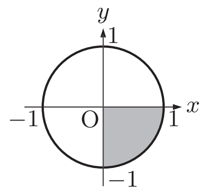
【1】(1) 第 2 象限



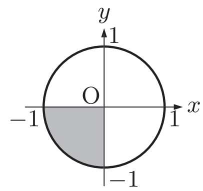
(2) 第 3 象限



(3) 第 4 象限



(4) 第 3 象限



【2】

θ	0°	30°	45°	60°	90°	120°	135°	150°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$

θ	180°	210°	225°	240°	270°	300°	315°	330°	360°
$\sin\theta$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos\theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

【3】(1)

$$\cos 120^\circ \tan 315^\circ + \sin 90^\circ \cos 180^\circ = \left(-\frac{1}{2}\right) \times (-1) + 1 \times (-1) = \frac{1}{2} - 1 = -\frac{1}{2}$$

(2)

$$\cos 240^\circ \sin 60^\circ - \sin 300^\circ \cos 120^\circ = \left(-\frac{1}{2}\right) \times \frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) \times \left(-\frac{1}{2}\right) = -\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$$

(3)

$$\sin 120^\circ \sin 315^\circ - \sin 240^\circ \cos 150^\circ$$
$$= \frac{\sqrt{3}}{2} \times \left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right) \times \left(-\frac{\sqrt{3}}{2}\right)$$
$$= -\frac{\sqrt{6}}{4} - \frac{3}{4} = -\frac{\sqrt{6}+3}{4}$$

(4)
$$\cos 315^\circ - \cos 150^\circ \tan 30^\circ + \sin 210^\circ - \sin 135^\circ$$
$$= \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2}\right) \times \frac{\sqrt{3}}{3} + \left(-\frac{1}{2}\right) - \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{1}{2} - \frac{\sqrt{2}}{2} = \mathbf{0}$$

【4】(1) $\sin\theta = \dfrac{1}{2}$

$\theta = \mathbf{30^\circ}$**，**$\mathbf{150^\circ}$

(2) $\cos\theta = \dfrac{\sqrt{3}}{2}$

$\theta = \mathbf{30^\circ}$**，**$\mathbf{330^\circ}$

(3) $\tan\theta = \sqrt{3}$

$\theta = \mathbf{60^\circ}$**，**$\mathbf{240^\circ}$

(4) $2\sin\theta + \sqrt{2} = 0$

$\sin\theta = -\dfrac{\sqrt{2}}{2}$

$\theta = \mathbf{225^\circ}$**，**$\mathbf{315^\circ}$

(5) $2\cos\theta + \sqrt{3} = 0$

$\cos\theta = -\dfrac{\sqrt{3}}{2}$

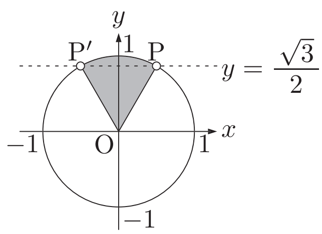
$\theta = \mathbf{150^\circ}$**，**$\mathbf{210^\circ}$

(6) $\tan\theta = 0$

$\theta = \mathbf{0^\circ}$**，**$\mathbf{180^\circ}$

【5】(1) $\sin\alpha = \dfrac{\sqrt{3}}{2}$ を満たす α の値は，単位円周上の y 座標が $\dfrac{\sqrt{3}}{2}$ である点 P，P′ に対応する角であるから，

$\alpha = 60^\circ$，120°

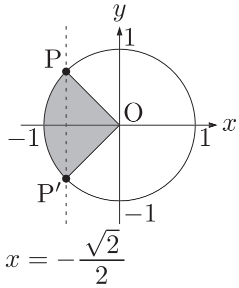


y 座標が $\dfrac{\sqrt{3}}{2}$ より大きい単位円の周上の点の集合は，$\overset{\frown}{\mathrm{PP'}}$ であるから，求める θ の値の範囲は

$\mathbf{60^\circ < \theta < 120^\circ}$

(2) $\cos\alpha = -\dfrac{\sqrt{2}}{2}$ を満たす α の値は単位円周上に x 座標が $-\dfrac{\sqrt{2}}{2}$ である点 P，P′ に対応する角であるから

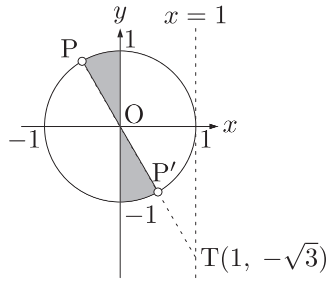
$\alpha = 135^\circ$，225°



単位円周上の x 座標が $-\dfrac{\sqrt{2}}{2}$ 以下の点の集合は，$\overset{\frown}{\mathrm{PP'}}$ であるから，求める θ の値の範囲は

$\mathbf{135^\circ \leqq \theta \leqq 225^\circ}$

(3) $\tan\alpha = -\sqrt{3}$ を満たす α の値は，直線 $x=1$ 上での y 座標が $-\sqrt{3}$ である点 T に対する単位円周上の点 P，P′ に対応する角であるから

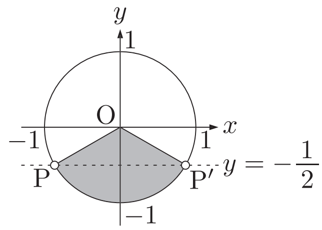


$$\alpha = 120^\circ,\ 300^\circ$$

直線 $x=1$ 上での y 座標が $-\sqrt{3}$ より小さい点の集合を考えて

$$\mathbf{90^\circ < \boldsymbol{\theta} < 120^\circ,\ 270^\circ < \boldsymbol{\theta} < 300^\circ}$$

(4)



$$2\sin\theta + 1 < 0$$
$$\sin\theta < -\frac{1}{2}$$

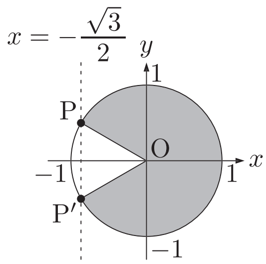
です．$\sin\alpha = -\frac{1}{2}$ を満たす α の値は，単位円周上の y 座標が $-\frac{1}{2}$ である点 P，P′ に対応する角であるから，

$$\alpha = 210^\circ,\ 330^\circ$$

y 座標が $-\frac{1}{2}$ より小さい点の集合は，$\overset{\frown}{\mathrm{PP'}}$ であるから，求める θ の値の範囲は

$$\mathbf{210^\circ < \boldsymbol{\theta} < 330^\circ}$$

(5)



$$2\cos\theta \geqq -\sqrt{3}$$
$$\cos\theta \geqq -\frac{\sqrt{3}}{2}$$

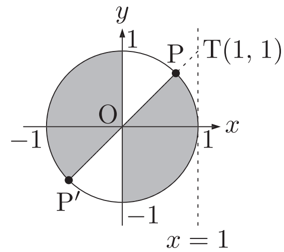
です．$\cos\alpha = -\frac{\sqrt{3}}{2}$ を満たす α の値は単位円周上に x 座標が $-\frac{\sqrt{3}}{2}$ である点 P，P′ に対応する角であるから

$$\alpha = 150^\circ,\ 210^\circ$$

単位円周上の x 座標が $-\frac{\sqrt{3}}{2}$ 以上の点の集合は，$\overset{\frown}{\mathrm{AP}}$ と $\overset{\frown}{\mathrm{P'A}}$ であるから，求める θ の値の範囲は

$$\mathbf{0^\circ \leqq \boldsymbol{\theta} \leqq 150^\circ,\ 210^\circ \leqq \boldsymbol{\theta} < 360^\circ}$$

(6) $\tan\alpha = 1$ を満たす α の値は，直線 $x=1$ 上での y 座標が 1 である点 T に対する単位円周上の点 P，P′ に対応する角であるから



$$\alpha = 45°,\ 225°$$

直線 $x=1$ 上での y 座標が 1 以下の点の集合を考えて

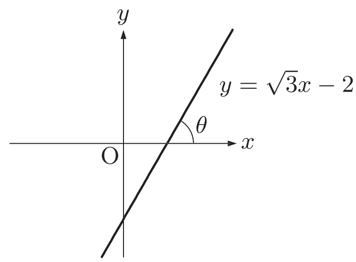
$$\boldsymbol{0° \leqq \theta \leqq 45°,\ 90° < \theta \leqq 225°,\ 270° < \theta < 360°}$$

【6】(1) 直線 $y=\sqrt{3}x-2$ と x 軸の正の向きとのなす角を
$\theta\ (0^\circ \leqq \theta \leqq 180^\circ)$ とすると，

$$\tan\theta=\sqrt{3}$$

よって

$$\theta=\mathbf{60^\circ}$$

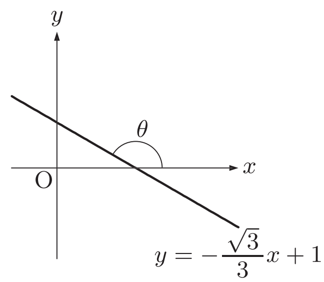


(2) 直線 $y=-\dfrac{\sqrt{3}}{3}x+1$ と x 軸の正の向きとのなす角を $\theta\ (0^\circ \leqq \theta \leqq 180^\circ)$ とすると，

$$\tan\theta=-\frac{\sqrt{3}}{3}$$

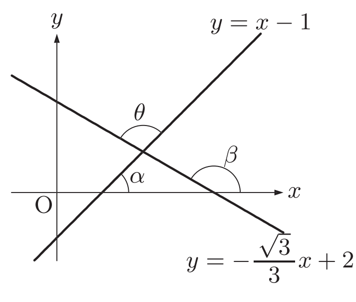
よって，

$$\theta=\mathbf{150^\circ}$$



(3) 直線 $y=x-1$ と x 軸の正の向きとのなす角を $\alpha\ (0^\circ \leqq \alpha \leqq 180^\circ)$,
直線 $y=-\dfrac{\sqrt{3}}{3}x+2$ と x 軸の正の向きとのなす角を $\beta\ (0^\circ \leqq \beta \leqq 180^\circ)$ とすると，

$$\tan\alpha=1,\ \tan\beta=-\frac{\sqrt{3}}{3}$$



ゆえに，

$$\alpha=45^\circ,\ \beta=150^\circ$$

2 直線のなす角を θ とすると，

$$\begin{aligned}\theta&=\beta-\alpha\\&=150^\circ-45^\circ\\&=105^\circ>90^\circ\end{aligned}$$

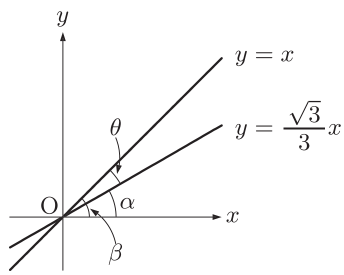
求める角は鋭角なので，

$$180^\circ-105^\circ=\mathbf{75^\circ}$$

(4) $x-\sqrt{3}y=0$ より，$y=\dfrac{\sqrt{3}}{3}x$

$x-y=0$ より，$y=x$

直線 $y=\dfrac{\sqrt{3}}{3}x$ と x 軸の正の向きとのなす角を α，直線 $y=x$ と x 軸の正の向きとのなす角を β とすると，



$$\tan\alpha=\frac{\sqrt{3}}{3},\ \tan\beta=1$$

ゆえに，

$$\alpha=30^\circ,\ \beta=45^\circ$$

2 直線のなす角を θ とすると，

$$\begin{aligned}\theta&=\beta-\alpha\\&=45^\circ-30^\circ\\&=15^\circ<90^\circ\end{aligned}$$

よって，求める鋭角は，**15°**

(5) 求める直線を $y=ax+b\cdots$ ① とおく．

$y=ax+b$ と x 軸の正の向きとのなす角が 135° なので，

$$a=\tan 135^\circ$$
$$a=-1$$

①に，$a=-1$ を代入すると，

$$y=-x+b$$

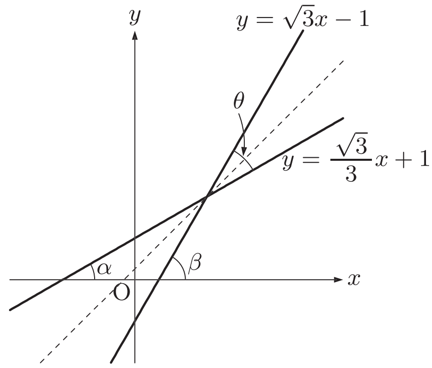
この直線は $(-1,\ 2)$ を通るので，

$$2=1+b$$
$$b=1$$

よって，$\boldsymbol{y=-x+1}$

(6) まず 2 直線のなす角を求める．

直線 $y = \dfrac{\sqrt{3}}{3}x + 1$ と x 軸の正の向きとのなす角を α $(0° \leqq \alpha \leqq 180°)$，直線 $y = \sqrt{3}x - 1$ と x 軸の正の向きとのなす角を β $(0° \leqq \beta \leqq 180°)$ とすると，



$$\tan\alpha = \frac{\sqrt{3}}{3},\ \tan\beta = \sqrt{3}$$

ゆえに

$$\alpha = 30°,\ \beta = 60°$$

2 直線のなす角を θ とすると，

$$\begin{aligned}\theta &= \beta - \alpha\\ &= 60° - 30°\\ &= 30°\end{aligned}$$

よって，2 直線のなす鋭角は $30°$ である．
求める直線はこれを 2 等分するので

$$30° \div 2 = 15°$$

つまり $y = \dfrac{\sqrt{3}}{3}x + 1$ と求める直線のなす角は $15°$ なので，求める直線と x 軸のなす角を γ とすると

$$\gamma = 30° + 15° = 45°$$
$$\tan\gamma = 1$$

よって，求める直線を $y = ax + b$ とすると

$$a = 1$$

また，求める直線は $y = \dfrac{\sqrt{3}}{3}x + 1,\ y = \sqrt{3}x - 1$ の交点を通る．
交点は

$$(x,\ y) = (\sqrt{3},\ 2)$$

より，$y = x + b$ に代入して

$$2 = \sqrt{3} + b$$
$$b = 2 - \sqrt{3}$$

したがって

$$\boldsymbol{y = x + 2 - \sqrt{3}}$$

添削課題

【1】P(x, y), $\mathrm{OP}=\sqrt{x^2+y^2}=r$ とおくと,

$$\sin\theta=\frac{y}{r},\ \cos\theta=\frac{x}{r},\ \tan\theta=\frac{y}{x}$$

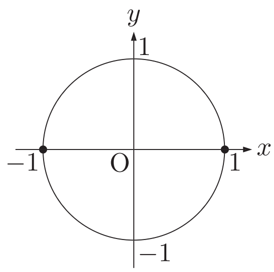
(1) $r=17$ だから,
$$\boldsymbol{\sin\theta=\frac{8}{17},\ \cos\theta=\frac{15}{17},\ \tan\theta=\frac{8}{15}}$$

(2) $r=41$ だから,
$$\boldsymbol{\sin\theta=\frac{40}{41},\ \cos\theta=-\frac{9}{41},\ \tan\theta=-\frac{40}{9}}$$

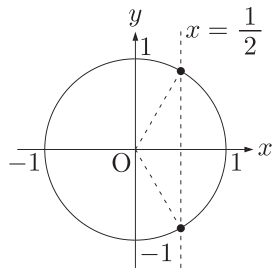
(3) $r=29$ だから,
$$\boldsymbol{\sin\theta=-\frac{21}{29},\ \cos\theta=-\frac{20}{29},\ \tan\theta=\frac{21}{20}}$$

(4) $r=13$ だから,
$$\boldsymbol{\sin\theta=-\frac{5}{13},\ \cos\theta=\frac{12}{13},\ \tan\theta=-\frac{5}{12}}$$

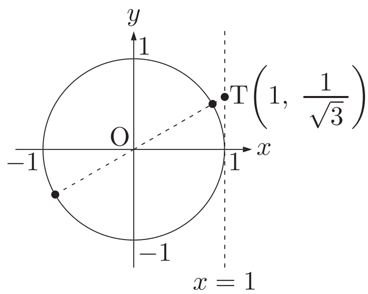
【2】(1) $\boldsymbol{\theta=0^\circ,\ 180^\circ}$



(2) $\boldsymbol{\theta=60^\circ,\ 300^\circ}$

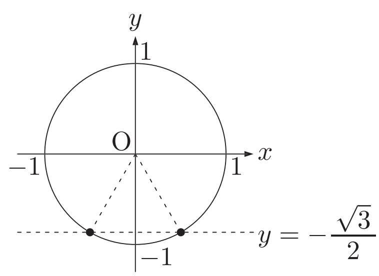


(3) $\boldsymbol{\theta=30^\circ,\ 210^\circ}$

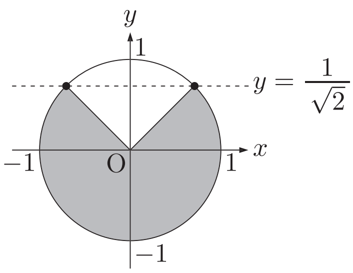


(4) $2\sin\theta+\sqrt{3}=0$ より, $\sin\theta=-\frac{\sqrt{3}}{2}$

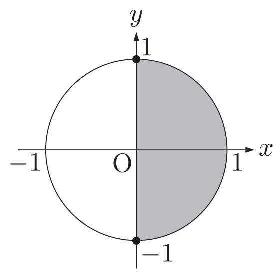
よって, $\boldsymbol{\theta=240^\circ,\ 300^\circ}$



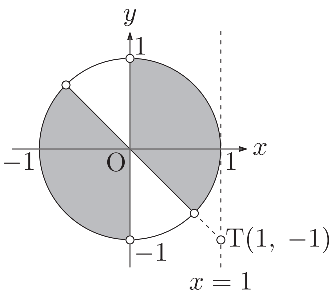
【3】(1) $\mathbf{0^\circ \leqq \theta \leqq 45^\circ,\ 135^\circ \leqq \theta < 360^\circ}$



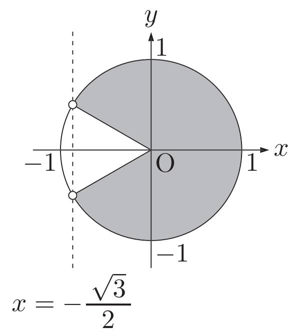
(2) $\mathbf{0^\circ \leqq \theta \leqq 90^\circ,\ 270^\circ \leqq \theta < 360^\circ}$



(3) $\mathbf{0^\circ \leqq \theta < 90^\circ,\ 135^\circ < \theta < 270^\circ,}$
$\mathbf{315^\circ < \theta < 360^\circ}$



(4) $2\cos\theta + \sqrt{3} > 0$ より，$\cos\theta > -\frac{\sqrt{3}}{2}$
よって，
$\mathbf{0^\circ \leqq \theta < 150^\circ,\ 210^\circ < \theta < 360^\circ}$



【4】(1) 直線 $y = -\sqrt{3}x + 5$ と x 軸の正の向きのなす角を θ とすると，
$\tan\theta = -\sqrt{3}$
$\theta = 120^\circ$ ($0^\circ \leqq \theta < 180^\circ$ より)
よって，**120°**

(2) 直線 $y = -\frac{1}{\sqrt{3}}x + 3$ と x 軸の正の向きとのなす角を α
直線 $y = -x - 2$ と x 軸の正の向きとのなす角を β
とすると，
$\tan\alpha = -\frac{1}{\sqrt{3}}$
$\therefore\ \alpha = 150^\circ$ ($0^\circ \leqq \alpha < 180^\circ$ より)
一方，
$\tan\beta = -1$
$\therefore\ \beta = 135^\circ$ ($0^\circ \leqq \beta < 180^\circ$ より)
よって，なす角 θ は，
$\theta = \alpha - \beta$
$= \mathbf{15^\circ}$ $(< 90^\circ)$

２３章　三角比（３）ー式の値ー

問題

【1】(1)

$$
\begin{aligned}
&\cos 40^\circ + \cos 80^\circ + \cos 100^\circ + \cos 140^\circ \\
=&\cos 40^\circ + \cos 80^\circ + \cos(180^\circ - 80^\circ) + \cos(180^\circ - 40^\circ) \\
=&\cos 40^\circ + \cos 80^\circ - \cos 80^\circ - \cos 40^\circ \\
=&\mathbf{0}
\end{aligned}
$$

(2)

$$
\begin{aligned}
&\sin 200^\circ \cos 160^\circ + \cos 200^\circ \sin 160^\circ \\
=&\sin(180^\circ + 20^\circ)\cos(180^\circ - 20^\circ) + \cos(180^\circ + 20^\circ)\sin(180^\circ - 20^\circ) \\
=&(-\sin 20^\circ) \times (-\cos 20^\circ) + (-\cos 20^\circ) \times \sin 20^\circ \\
=&\sin 20^\circ \cos 20^\circ - \sin 20^\circ \cos 20^\circ \\
=&\mathbf{0}
\end{aligned}
$$

(3)

$$
\begin{aligned}
\sin^2 25^\circ + \sin^2 65^\circ &= \sin^2 25^\circ + \sin^2(90^\circ - 25^\circ) \\
&= \sin^2 25^\circ + \cos^2 25^\circ \\
&= \mathbf{1}
\end{aligned}
$$

(4)

$$
\begin{aligned}
&(\sin 20^\circ + \cos 20^\circ)^2 + (\sin 110^\circ + \cos 110^\circ)^2 \\
=&(\sin 20^\circ + \cos 20^\circ)^2 + \{\sin(90^\circ + 20^\circ) + \cos(90^\circ + 20^\circ)\}^2 \\
=&(\sin 20^\circ + \cos 20^\circ)^2 + (\cos 20^\circ - \sin 20^\circ)^2 \\
=&(\sin^2 20^\circ + 2\sin 20^\circ \cos 20^\circ + \cos^2 20^\circ) \\
&\qquad + (\cos^2 20^\circ - 2\sin 20^\circ \cos 20^\circ + \sin^2 20^\circ) \\
=&(1 + 2\sin 20^\circ \cos 20^\circ) + (1 - 2\sin 20^\circ \cos 20^\circ) \\
=&\mathbf{2}
\end{aligned}
$$

(5)

$$
\begin{aligned}
\frac{1}{\sin^2 10^\circ} - \tan^2 100^\circ &= \frac{1}{\sin^2 10^\circ} - \tan^2(90^\circ + 10^\circ) \\
&= \frac{1}{\sin^2 10^\circ} - \left(-\frac{1}{\tan 10^\circ}\right)^2 \\
&= \frac{1}{\sin^2 10^\circ} - \frac{1}{\tan^2 10^\circ} \\
&= \frac{1}{\sin^2 10^\circ} - \frac{\cos^2 10^\circ}{\sin^2 10^\circ} \\
&= \frac{1 - \cos^2 10^\circ}{\sin^2 10^\circ} \\
&= \frac{\sin^2 10^\circ}{\sin^2 10^\circ} \\
&= \mathbf{1}
\end{aligned}
$$

【2】(1)

$$
\begin{aligned}
&\sin(180^\circ-\theta)\cos(180^\circ-\theta)+\cos(90^\circ-\theta)\sin(90^\circ-\theta)\\
=&\sin\theta\times(-\cos\theta)+\sin\theta\times\cos\theta\\
=&\mathbf{0}
\end{aligned}
$$

(2)

$$
\begin{aligned}
\tan(90^\circ-\theta)\tan(180^\circ-\theta)&=\frac{1}{\tan\theta}\times(-\tan\theta)\\
&=\mathbf{-1}
\end{aligned}
$$

(3)

$$
\begin{aligned}
&\sin\theta+\sin(90^\circ+\theta)+\sin(180^\circ+\theta)+\sin(270^\circ+\theta)\\
=&\sin\theta+\cos\theta-\sin\theta+\sin\{180^\circ+(90^\circ+\theta)\}\\
=&\sin\theta+\cos\theta-\sin\theta-\sin(90^\circ+\theta)\\
=&\sin\theta+\cos\theta-\sin\theta-\cos\theta\\
=&\mathbf{0}
\end{aligned}
$$

(4)

$$
\begin{aligned}
&\sin(90^\circ-\theta)\sin(180^\circ+\theta)+\sin(180^\circ-\theta)\sin(90^\circ+\theta)\\
=&\cos\theta\times(-\sin\theta)+\sin\theta\times\cos\theta\\
=&-\sin\theta\cos\theta+\sin\theta\cos\theta\\
=&\mathbf{0}
\end{aligned}
$$

【3】(1) $\sin^2\theta+\cos^2\theta=1$ より，

$$
\begin{aligned}
\left(\frac{\sqrt{5}}{3}\right)^2+\cos^2\theta&=1\\
\frac{5}{9}+\cos^2\theta&=1\\
\cos^2\theta&=\frac{4}{9}\\
\cos\theta&=\pm\frac{2}{3}
\end{aligned}
$$

θ が第 1 象限の角だから，$\cos\theta>0$ なので，

$$\cos\theta=\frac{2}{3}$$

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ より，

$$\tan\theta=\frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}}=\frac{\sqrt{5}}{2}$$

よって，

$$\boldsymbol{\cos\theta=\frac{2}{3},\ \tan\theta=\frac{\sqrt{5}}{2}}$$

(2) $\sin^2\theta+\cos^2\theta=1$ より，

$$
\begin{aligned}
\sin^2\theta+\left(\frac{\sqrt{2}}{2}\right)^2&=1\\
\sin^2\theta+\frac{2}{4}&=1\\
\sin^2\theta&=\frac{1}{2}\\
\sin\theta&=\pm\frac{1}{\sqrt{2}}\\
&=\pm\frac{\sqrt{2}}{2}
\end{aligned}
$$

θ が第 1 象限の角だから，$\sin\theta>0$ なので，

$$\sin\theta=\frac{\sqrt{2}}{2}$$

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ より，

$$\tan\theta=\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}=\mathbf{1}$$

よって，

$$\boldsymbol{\sin\theta=\frac{\sqrt{2}}{2},\ \tan\theta=1}$$

(3) $\sin^2\theta + \cos^2\theta = 1$ より,

$$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$$
$$\frac{9}{25} + \cos^2\theta = 1$$
$$\cos^2\theta = \frac{16}{25}$$
$$\cos\theta = \pm\frac{4}{5}$$

θ は第 2 象限の角だから, $\cos\theta < 0$ なので,

$$\cos\theta = -\frac{4}{5}$$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ より,

$$\tan\theta = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}$$

よって,

$$\boldsymbol{\cos\theta = -\frac{4}{5},\ \tan\theta = -\frac{3}{4}}$$

(4) $1 + \tan^2\theta = \dfrac{1}{\cos^2\theta}$ より,

$$1 + \left(-\frac{1}{2}\right)^2 = \frac{1}{\cos^2\theta}$$
$$\frac{5}{4} = \frac{1}{\cos^2\theta}$$
$$\cos^2\theta = \frac{4}{5}$$
$$\cos\theta = \pm\frac{2\sqrt{5}}{5}$$

θ が第 2 象限の角だから, $\cos\theta < 0$ なので,

$$\cos\theta = -\frac{2\sqrt{5}}{5}$$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ より,

$$-\frac{1}{2} = \frac{\sin\theta}{-\frac{2\sqrt{5}}{5}}$$
$$\sin\theta = \frac{\sqrt{5}}{5}$$

よって,

$$\boldsymbol{\cos\theta = -\frac{2\sqrt{5}}{5},\ \sin\theta = \frac{\sqrt{5}}{5}}$$

(5) $\sin^2\theta+\cos^2\theta=1$ より,

$$\sin^2\theta+\left(-\frac{12}{13}\right)^2=1$$
$$\sin^2\theta+\frac{144}{169}=1$$
$$\sin^2\theta=\frac{25}{169}$$
$$\sin\theta=\pm\frac{5}{13}$$

θ が第 3 象限の角だから，$\sin\theta<0$ なので,

$$\sin\theta=-\frac{5}{13}$$

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ より,

$$\tan\theta=\frac{-\frac{5}{13}}{-\frac{12}{13}}=\frac{5}{12}$$

よって,

$$\boldsymbol{\sin\theta=-\frac{5}{13},\ \tan\theta=\frac{5}{12}}$$

(6) $1+\tan^2\theta=\dfrac{1}{\cos^2\theta}$ より,

$$1+\left(\frac{1}{5}\right)^2=\frac{1}{\cos^2\theta}$$
$$\frac{26}{25}=\frac{1}{\cos^2\theta}$$
$$\cos^2\theta=\frac{25}{26}$$
$$\cos\theta=\pm\frac{5}{\sqrt{26}}=\pm\frac{5\sqrt{26}}{26}$$

θ が第 3 象限の角だから，$\cos\theta<0$ なので,

$$\cos\theta=-\frac{5\sqrt{26}}{26}$$

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ より,

$$\frac{1}{5}=\frac{\sin\theta}{-\frac{5\sqrt{26}}{26}}$$
$$\sin\theta=-\frac{\sqrt{26}}{26}$$

よって,

$$\boldsymbol{\cos\theta=-\frac{5\sqrt{26}}{26},\ \sin\theta=-\frac{\sqrt{26}}{26}}$$

(7) $\sin^2\theta+\cos^2\theta=1$ より，

$$\left(-\frac{1}{3}\right)^2+\cos^2\theta=1$$
$$\frac{1}{9}+\cos^2\theta=1$$
$$\cos^2\theta=\frac{8}{9}$$
$$\cos\theta=\pm\frac{2\sqrt{2}}{3}$$

θ が第 4 象限の角だから，$\cos\theta>0$ なので，

$$\cos\theta=\frac{2\sqrt{2}}{3}$$

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ より，

$$\tan\theta=\frac{-\frac{1}{3}}{\frac{2\sqrt{2}}{3}}=-\frac{1}{2\sqrt{2}}$$
$$=-\frac{\sqrt{2}}{4}$$

よって，

$$\boldsymbol{\cos\theta=\frac{2\sqrt{2}}{3},\ \tan\theta=-\frac{\sqrt{2}}{4}}$$

(8) $1+\tan^2\theta=\dfrac{1}{\cos^2\theta}$ より，

$$1+(-2)^2=\frac{1}{\cos^2\theta}$$
$$5=\frac{1}{\cos^2\theta}$$
$$\cos^2\theta=\frac{1}{5}$$
$$\cos\theta=\pm\frac{\sqrt{5}}{5}$$

θ が第 4 象限の角だから，$\cos\theta>0$ なので，

$$\cos\theta=\frac{\sqrt{5}}{5}$$

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ より，

$$-2=\frac{\sin\theta}{\frac{\sqrt{5}}{5}}\qquad\sin\theta=-\frac{2\sqrt{5}}{5}$$

よって，

$$\boldsymbol{\sin\theta=-\frac{2\sqrt{5}}{5},\ \cos\theta=\frac{\sqrt{5}}{5}}$$

【4】(1) $\sin^2\theta + \cos^2\theta = 1$ より,

$$\sin^2\theta + \left(\frac{3}{4}\right)^2 = 1$$
$$\sin^2\theta = \frac{7}{16}$$
$$\sin\theta = \pm\frac{\sqrt{7}}{4}$$

$0^\circ \leqq \theta \leqq 180^\circ$ より, $\sin\theta = \dfrac{\sqrt{7}}{4}$ よって,

$$\begin{aligned}
&\frac{\sin\theta}{1+\cos\theta} + \frac{1}{\tan\theta} \\
=&\frac{\sin\theta}{1+\cos\theta} + \frac{\cos\theta}{\sin\theta} \\
=&\frac{\sin^2\theta + (1+\cos\theta)\cos\theta}{(1+\cos\theta)\sin\theta} \\
=&\frac{\sin^2\theta + \cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta} \\
=&\frac{(\sin^2\theta + \cos^2\theta) + \cos\theta}{(1+\cos\theta)\sin\theta} \\
=&\frac{1+\cos\theta}{(1+\cos\theta)\sin\theta} \\
=&\frac{1}{\sin\theta} = \frac{1}{\frac{\sqrt{7}}{4}} \\
=&\boldsymbol{\frac{4\sqrt{7}}{7}}
\end{aligned}$$

(2) $\sin^2\theta + \cos^2\theta = 1$ より,

$$\sin^2\theta + \left(\frac{5}{13}\right)^2 = 1$$
$$\sin^2\theta = \frac{144}{169}$$
$$\sin\theta = \pm\frac{12}{13}$$

$0^\circ \leqq \theta \leqq 180^\circ$ より, $\sin\theta = \dfrac{12}{13}$ よって,

$$\begin{aligned}
&\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} \\
=&\frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}} \\
=&\frac{24-15}{48-45} \\
=&\frac{9}{3} = \mathbf{3}
\end{aligned}$$

(3)

$$\begin{aligned}
\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} &= \frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} \\
&= \frac{2}{1-\sin^2\theta} \\
&= \frac{2}{\cos^2\theta} \\
&= 2(1+\tan^2\theta) \quad \left(\because \quad 1+\tan^2\theta = \frac{1}{\cos^2\theta}\right) \\
&= 2\{1+(-2)^2\} \\
&= \mathbf{10}
\end{aligned}$$

(4)

$$\begin{aligned}
\left(\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}\right)^2 &= \left\{\frac{\cos^2\theta+(1+\sin\theta)^2}{(1+\sin\theta)\cos\theta}\right\}^2\\
&= \left\{\frac{\cos^2\theta+1+2\sin\theta+\sin^2\theta}{(1+\sin\theta)\cos\theta}\right\}^2\\
&= \left\{\frac{(\cos^2\theta+\sin^2\theta)+1+2\sin\theta}{(1+\sin\theta)\cos\theta}\right\}^2\\
&= \left\{\frac{1+1+2\sin\theta}{(1+\sin\theta)\cos\theta}\right\}^2\\
&= \left\{\frac{2+2\sin\theta}{(1+\sin\theta)\cos\theta}\right\}^2\\
&= \left\{\frac{2(1+\sin\theta)}{(1+\sin\theta)\cos\theta}\right\}^2\\
&= \left(\frac{2}{\cos\theta}\right)^2\\
&= \frac{4}{\cos^2\theta}\\
&= 4(1+\tan^2\theta) \quad \left(\because\ 1+\tan^2\theta=\frac{1}{\cos^2\theta}\right)\\
&= 4\{1+(-3)^2\}\\
&= \mathbf{40}
\end{aligned}$$

【5】(1)

$$\sin\theta+\cos\theta=\frac{1}{2}$$

の両辺を平方すると

$$\begin{aligned}
(\sin\theta+\cos\theta)^2 &= \left(\frac{1}{2}\right)^2\\
1+2\sin\theta\cos\theta &= \frac{1}{4}\\
\sin\theta\cos\theta &= -\mathbf{\frac{3}{8}}
\end{aligned}$$

(2)

$$\begin{aligned}
&\sin^3\theta+\cos^3\theta\\
=&(\sin\theta+\cos\theta)^3\\
&\quad -3\sin\theta\cos\theta(\sin\theta+\cos\theta)\\
=&\left(\frac{1}{2}\right)^3-3\times\left(-\frac{3}{8}\right)\times\frac{1}{2}\\
=&\frac{1}{8}+\frac{9}{16}\\
=&\mathbf{\frac{11}{16}}
\end{aligned}$$

(3)

$$\begin{aligned}
&(\sin\theta-\cos\theta)^2\\
&=\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta\\
&=1-2\times\left(-\frac{3}{8}\right)\\
&=1+\frac{3}{4}=\frac{7}{4}
\end{aligned}$$

よって，$|\sin\theta-\cos\theta|\geqq 0$ より，

$$|\sin\theta-\cos\theta|=\mathbf{\frac{\sqrt{7}}{2}}$$

(4)

$$\begin{aligned}
\tan\theta+\frac{1}{\tan\theta} &= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\\
&= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\\
&= \frac{1}{\sin\theta\cos\theta}\\
&= \frac{1}{-\frac{3}{8}}\\
&= -\mathbf{\frac{8}{3}}
\end{aligned}$$

【6】(1)

$$\sin\theta-\cos\theta=\frac{5}{4}$$

の両辺を平方すると

$$(\sin\theta-\cos\theta)^2=\left(\frac{5}{4}\right)^2$$
$$1-2\sin\theta\cos\theta=\frac{25}{16}$$
$$\sin\theta\cos\theta=-\mathbf{\frac{9}{32}}$$

(2)

$$\begin{aligned}&\sin^3\theta-\cos^3\theta\\ =&(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)\\ =&(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)\\ =&\frac{5}{4}\times\left(1-\frac{9}{32}\right)\\ =&\frac{5}{4}\times\frac{23}{32}\\ =&\mathbf{\frac{115}{128}}\end{aligned}$$

(3)

$$\begin{aligned}&(\sin\theta+\cos\theta)^2\\ &=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta\\ &=1+2\sin\theta\cos\theta\\ &=1+2\times\left(-\frac{9}{32}\right)\\ &=\frac{7}{16}\end{aligned}$$

なので，　$\sin\theta+\cos\theta=\pm\frac{\sqrt{7}}{4}$

ここで，(1) より，$\sin\theta\cos\theta<0$.

$0°<\theta<135°$ より，$90°<\theta<135°$.

これより，

$$\frac{\sqrt{2}}{2}<\sin\theta<1,\ -\frac{\sqrt{2}}{2}<\cos\theta<0$$

であるので，　$\sin\theta+\cos\theta>0$

よって，　$\sin\theta+\cos\theta=\mathbf{\frac{\sqrt{7}}{4}}$

(4)

$$\begin{aligned}\tan\theta+\frac{1}{\tan\theta}&=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\\ &=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\\ &=\frac{1}{\sin\theta\cos\theta}\\ &=\frac{1}{-\frac{9}{32}}\\ &=-\mathbf{\frac{32}{9}}\end{aligned}$$

【7】(1)

$$\begin{aligned}(\text{左辺}) &= \frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}\\ &= \frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}\\ &= \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}\\ &= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}\\ &= \frac{2}{\cos\theta} = (\text{右辺})\end{aligned}$$

よって，等式は成立する．（証明終）

(2)

$$\begin{aligned}(\text{左辺}) &= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}\\ &= \frac{\sin\theta}{\cos\theta} \div \frac{1}{\cos^2\theta}\\ &= \sin\theta\cos\theta = (\text{右辺})\end{aligned}$$

よって，等式は成立する．（証明終）

(3)

$$\begin{aligned}(\text{左辺}) &= \sin^2\theta - (\sin^2\theta)^2\\ &= (1-\cos^2\theta)-(1-\cos^2\theta)^2\\ &= (1-\cos^2\theta)-(1-2\cos^2\theta+\cos^4\theta)\\ &= 1-\cos^2\theta-1+2\cos^2\theta-\cos^4\theta\\ &= \cos^2\theta-\cos^4\theta = (\text{右辺})\end{aligned}$$

よって，等式は成立する．（証明終）

(4)

$$\begin{aligned}(\text{左辺}) &= (1+\tan^2\theta)(1-\tan^2\theta)\cos^2\theta+\tan^2\theta\\ &= \frac{1}{\cos^2\theta}\times(1-\tan^2\theta)\times\cos^2\theta+\tan^2\theta\\ &= 1-\tan^2\theta+\tan^2\theta\\ &= 1 = (\text{右辺})\end{aligned}$$

よって，等式は成立する．（証明終）

【8】(1)

$$
\begin{aligned}
(\text{右辺}) &= \left(\frac{1+\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\sin\theta}{\cos\theta}}\right)^2 \\
&= \left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)^2 \\
&= \frac{\cos^2\theta+2\cos\theta\sin\theta+\sin^2\theta}{\cos^2\theta-2\cos\theta\sin\theta+\sin^2\theta} \\
&= \frac{1+2\sin\theta\cos\theta}{1-2\sin\theta\cos\theta} = (\text{左辺})
\end{aligned}
$$

よって，等式は成立する．（証明終）

(2)

$$
\begin{aligned}
(\text{左辺}) &= \frac{2\cos^2\theta-(\cos^2\theta+\sin^2\theta)}{(\sin^2\theta+\cos^2\theta)-2\sin\theta\cos\theta} \\
&= \frac{\cos^2\theta-\sin^2\theta}{(\cos\theta-\sin\theta)^2} \\
&= \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta-\sin\theta)^2} \\
&= \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}
\end{aligned}
$$

一方，

$$
\begin{aligned}
(\text{右辺}) &= \frac{(\sin^2\theta+\cos^2\theta)+2\sin\theta\cos\theta}{(\sin^2\theta+\cos^2\theta)-2\sin^2\theta} \\
&= \frac{(\sin\theta+\cos\theta)^2}{\cos^2\theta-\sin^2\theta} \\
&= \frac{(\sin\theta+\cos\theta)^2}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)} \\
&= \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}
\end{aligned}
$$

よって，(左辺) = (右辺) より，等式は成立する．（証明終）

添削課題

【1】(1)

$$(\sin 35^\circ + \cos 35^\circ)^2 + (\sin 55^\circ - \cos 55^\circ)^2$$
$$=(\sin 35^\circ + \cos 35^\circ)^2 + (\cos 35^\circ - \sin 35^\circ)^2$$
$$=2(\sin^2 35^\circ + \cos^2 35^\circ)$$
$$=2 \cdot 1 = \mathbf{2}$$

(2)

$$\cos 80^\circ = \sin 10^\circ$$
$$\sin 170^\circ = \sin 10^\circ$$
$$\sin 260^\circ = -\sin 80^\circ = -\cos 10^\circ$$
$$\cos 350^\circ = -\cos 170^\circ = \cos 10^\circ$$

だから，

$$\cos 80^\circ \sin 170^\circ - \sin 260^\circ \cos 350^\circ = \sin^2 10^\circ + \cos^2 10^\circ = \mathbf{1}$$

【2】(1)

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(-\frac{1}{3}\right)^2 = \frac{8}{9}$$

$180^\circ \leqq \theta \leqq 270^\circ$ のとき，$-1 \leqq \sin\theta \leqq 0$ だから，

$$\sin\theta = -\boldsymbol{\frac{2\sqrt{2}}{3}}$$

よって，

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{2\sqrt{2}}{3}}{-\frac{1}{3}} = \mathbf{2\sqrt{2}}$$

(2) $\cos\theta = -\dfrac{1}{3}$ $(0^\circ \leqq \theta \leqq 180^\circ)$ のとき，

θ は鈍角で，

$$\sin\theta = \frac{2\sqrt{2}}{3},\ \tan\theta = -2\sqrt{2}$$

だから，

$$\tan\theta - \frac{\cos(180^\circ - \theta)}{1 + \sin(180^\circ - \theta)}$$
$$=\tan\theta + \frac{\cos\theta}{1 + \sin\theta}$$
$$=-2\sqrt{2} + \frac{-\frac{1}{3}}{1 + \frac{2\sqrt{2}}{3}}$$
$$=-2\sqrt{2} - \frac{1}{3 + 2\sqrt{2}}$$
$$=-2\sqrt{2} - (3 - 2\sqrt{2})$$
$$=\mathbf{-3}$$

＜別解＞

$$（与式）= \tan\theta + \frac{\cos\theta}{1 + \sin\theta}$$
$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta}$$
$$= \frac{\sin\theta(1 + \sin\theta) + \cos^2\theta}{\cos\theta(1 + \sin\theta)}$$
$$= \frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1 + \sin\theta)}$$
$$= \frac{\sin\theta + 1}{\cos\theta(1 + \sin\theta)}$$
$$= \frac{1}{\cos\theta} = \mathbf{-3}$$

【3】(1) $\sin\theta+\cos\theta=\dfrac{7}{5}$ の両辺を2乗にして，

$$1+2\sin\theta\cos\theta=\frac{49}{25}$$
$$\sin\theta\cos\theta=\frac{1}{2}\left(\frac{49}{25}-1\right)$$
$$\therefore\ \sin\theta\cos\theta=\boldsymbol{\frac{12}{25}}$$

(2) (1) より，

$$(\sin\theta-\cos\theta)^2=1-2\sin\theta\cos\theta$$
$$=1-\frac{24}{25}$$
$$=\frac{1}{25}$$
$$\therefore\ \sin\theta-\cos\theta=\pm\boldsymbol{\frac{1}{5}}$$

(3) $\sin\theta$ と $\cos\theta$ の和は $\dfrac{7}{5}$，積は $\dfrac{12}{25}$ だから，

$\sin\theta$，$\cos\theta$ は，2次方程式

$$x^2-\frac{7}{5}x+\frac{12}{25}=0$$

の解である．よって，

$$(\sin\theta,\ \cos\theta)=\left(\frac{3}{5},\ \frac{4}{5}\right),\ \left(\frac{4}{5},\ \frac{3}{5}\right)$$

よって，

$$\tan\theta=\frac{\sin\theta}{\cos\theta}=\boldsymbol{\frac{3}{4},\ \frac{4}{3}}$$

【4】$\cos\theta=x$，$\sin\theta=y$ とおくと，

$$(左辺)=\frac{1+y+x}{1+y-x}+\frac{1+y-x}{1+y+x}$$
$$=\frac{\{(1+y)+x\}^2+\{(1+y)-x\}^2}{\{(1+y)-x\}\{(1+y)+x\}}$$
$$=\frac{2\{(1+y)^2+x^2\}}{(1+y)^2-x^2}\quad\cdots(*)$$

ここで，$x^2+y^2=1$ より，$x^2=1-y^2$ だから，これを代入して，

$$(*)=\frac{2(1+2y+y^2+1-y^2)}{1+2y+y^2-(1-y^2)}$$
$$=\frac{4(1+y)}{2y(1+y)}$$
$$=\frac{2}{y}=\frac{2}{\sin\theta}$$
$$=(右辺)$$

よって，

$$\frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta}+\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\frac{2}{\sin\theta}$$

（証明終）

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